THE CENTRIFUGAL PUMP.

TURBINES AND WATER MOTORS:

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INCLUDING THE

THEORY AND PRACTICE OF HYDRAULICS.

(SPECIALLY ADAPTED FOR ENGINEERS.)

BY

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PREFACE.

THE matter comprised in the second edition of this book may be divided into seven parts: the first deals with general principles; the second with pressure engines producing rotary motion; the third with turbines whose power is obtained by altering the direction of motion and the velocity of the water; and I have added a description of the Pelton wheel, with some remarks on its theory.

In Chapter XXI. will be found descriptions of the various forms of steam turbines designed by the Hon. C. A. Parsons, to whom I am indebted for the whole of the matter and illustrations. I have endeavoured in Chapter XXII. to show that, in water turbines, theory and experiment agree very closely. Chapters XXIII. to XXX. inclusive deal with the centrifugal pump, and Chapters XXXI. and XXXII. with the fan. Chapter XXXIII. is a description of the hydraulic works at Niagara, and the book concludes with a short description of the hydraulic buffer stop. Graphical methods have been used wherever possible, but the advantage of trigonometry is so great, that it has been introduced wherever necessary.

In the preparation of the book no effort has been spared to make the text clear, by means of copious illustrations, wherever they were deemed desirable, and the matter will, it is thought, be found of special value to those engineering students preparing for the Honours Stages of the Science and Art and Technological Examinations in Machine Construction and Mechanical Engineering.

Rutherford College, CHAS. H. INNES.
December, 1898.

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CENTRIFUGAL PUMPS AND TURBINES.

CHAPTER I.

THE MOTION OF WATER UNDER PRESSURE CAUSED BY A GIVEN HEAD OF WATER.

FIG. 1 represents a vessel of water with an orifice A, which is at h feet below the surface, the water being always maintained at this level. Theory states that the velocity of flow through this orifice v in feet per second $= \sqrt{2\,g\,h}$ $= 8\sqrt{h}$, taking $g = 32$. The reasoning is as follows: A body falling a height h attains a velocity $= 8\sqrt{h}$, and, neglecting friction and cross motions, this is exactly what each particle of water does. The cause of the velocity in both cases is that gravity has done work $= w\,h$ foot-pounds on a weight of w lbs., which must, if unresisted, produce

kinetic energy $= \dfrac{w\,v^2}{2\,g}$ foot-pounds,

whence $\dfrac{w\,v^2}{2\,g} = w\,h$, and $v = 8\sqrt{h}$.

There are two reasons why this theory needs modification. If in the passage to A, fig. 1, enlarged in fig. 2, all the particles were flowing parallel to the sides of the pipe, the quantity of water passing any section C D would be A v cubic feet, where A = area of C D in square feet; but just before entrance into the pipe at E F the water is flowing in every direction, which causes an irregular motion of the particles, as shown at G H, reducing the velocity perpendicular to C D, and, in addition, wasting the energy by the friction of the particles rubbing against one another. This makes it necessary to introduce a coefficient of velocity C_v, so that the velocity of outflow is $C_v\,\sqrt{h}$.

Again, every orifice has a coefficient of contraction depending on its shape. The cause of the contraction is most noticeable at a sharp-edged orifice, fig. 3. As the water

but $h - h^1 = \frac{v^2}{2g}$, because h^1 is absorbed by friction, and $h - h^1$ produces the kinetic energy.

$$\therefore \frac{v^2}{2g\,C_v{}^2} - F\frac{v^2}{2g} = \frac{v^2}{2g}.$$

$$F = \left(\frac{1}{C_v{}^2} - 1\right)$$

CHAPTER II.

Measurement of the Power of a Stream.

The power of a stream may be used to drive machinery, and in order to know its magnitude we must measure the quantity of water flowing per second or per minute, and the

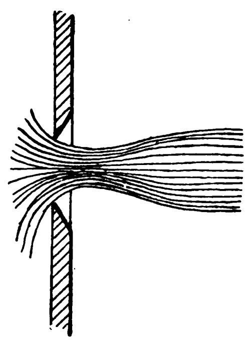

Fig 3.

distance it will fall—*i.e.*, the head. This latter is found by levelling, while the former may be calculated by the above formulæ if the stream is small, the water being made to flow through a number of completely immersed round or square sharp-edged orifices. It is, however, generally most convenient to form a notch in a temporary weir. The notch is generally rectangular or triangular in section; in the latter case the vertex being downwards (figs. 5, 6, 7), the sides and bottoms being chamfered; or, better, edged all

round with thin sheet iron, in order that contraction may not be suppressed, and the following formula may be applied:

For a rectangular notch—

$$Q = \tfrac{2}{3}\, c \,.\, b h \,.\, \sqrt{2 g h},$$
$$= 5{\cdot}35\, c b h \sqrt{h},$$

where Q = cubic feet per second;
b = breadth of notch;
h = height of surface of still water above the bottom of the notch;
c = a coefficient of discharge.

If b is one-fourth the width of weir, the least width advisable, $c = {\cdot}595$.

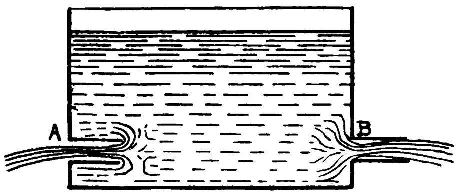

FIG. 4.

If b is the whole width of weir, $c = {\cdot}667$.

For any intermediate proportions—

$$c = {\cdot}57 + \frac{b}{10\,B},$$

where B = breadth of weir.

For the method of obtaining such a formula the reader is referred to Prof. Cotteril's "Applied Mechanics," page 450, section 236.

In consequence of variations in the coefficient of contraction already stated, which depend on the ratio $\frac{b}{B}$, and other variations which have been reduced to no general law, Prof. Thomson adopted a triangular notch, so that the issuing jet is always a similar figure—a triangle, with apex downwards. Here—

$$Q = \frac{8}{15}\, c\, \frac{b\, h}{2} \sqrt{2 g h}.$$

When $b = 2\,h, c = {\cdot}595, Q = 2{\cdot}54\, h^{\frac{5}{2}}$.
„ $b = 4\,h, c = {\cdot}620, Q = 5{\cdot}3\ h^{\frac{5}{2}}$.

In order to measure h, a scale must be driven in the ground in the pond above the notch, at a point where the water is sensibly still, or has a very slow motion. As the height h is liable to vary, it should be noted as often as possible.

When the stream is large, the area of the cross-section must be found, and the velocity must be measured at as many points as possible. The area should be divided into several parts, and the velocity in each part having been noted, the total quantity Q per second will be $A_1 v_1 + A_2 v_2$, &c., where A_1, A_2, &c., are the areas of the several parts,

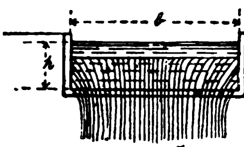

FIG 5.

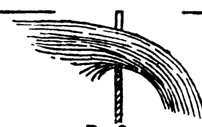

FIG 6

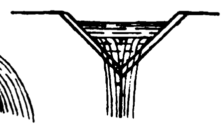

FIG 7.

and v_1, v_2, &c., the velocities therein. It will be less trouble, but not so accurate, to multiply the total area by the mean velocity. There are several instruments for measuring the velocity. The principle of all is as follows: A small revolving fan drives a spindle, on which is a screw which gives motion to a train of wheelwork, which, by means of pointers, records the number of revolutions. To graduate the instrument it must be drawn through still water at known velocities. It is fixed at the end of a pole, so that it can be placed at different depths in the stream whose velocity is to be measured.

CHAPTER III.

FORM ASSUMED BY THE ENERGY OF RISING OR FALLING WATER.

IF a stream of water flows continually without meeting any cause of loss of energy, such as friction of piping or sudden enlargements and contractions, a simple law may be used connecting the pressure, velocity, and head producing the flow. In fig. 8 a tank is shown, the surface of water being at a height H above a certain level, and the water is flowing through a pipe to work some machine or machines, let us suppose, the flow being unbroken. Then, neglecting

loss of energy at the orifice B and friction of pipe, we may assume that there is no loss of energy. The section of the pipe is variable, but the changes must be gradual. When first entering the tank each pound of water has H foot-pounds of potential energy, while at A it has only h foot-pounds of potential energy, the remainder of the energy being kinetic, and what we shall call pressure energy. Let v be the velocity, and p the pressure per square foot, 62·5 being the weight of a cubic foot of water; then the kinetic energy is $\frac{v^2}{2g}$, and we shall show that the pressure energy is $\frac{p}{62 \cdot 5}$ per lb.

Suppose we take a length of the pipe $= x$, by making x small enough we may neglect variations of h, p, and v. Also imagine a piston in the pipe separating the water above

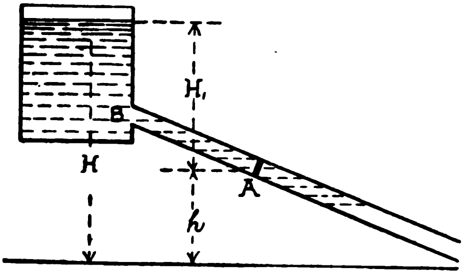

FIG 8.

from the water below. The water above, by means of its pressure, does work on the water below, and as the piston moves x feet, the work done is p A x foot-pounds, where A is the area of the pipe in square feet.

Thus, A x cubic feet of water do p A x foot-pounds;

∴ one cubic foot of water does p foot-pounds;

or, one pound of water does $\frac{p}{62 \cdot 5}$ foot-pounds.

∴ the total energy per pound, potential, pressure, and kinetic,

$$= h + \frac{p}{62 \cdot 5} + \frac{v^2}{2g}.$$

And, since we assume that there is no loss by friction, &c., this equals the initial potential energy per pound = H.

$$\therefore \text{H} = h + \frac{p}{62{\cdot}5} + \frac{v^2}{2g};$$

$$\therefore \text{H} - h = \frac{p}{62{\cdot}5} + \frac{v^2}{2g} = \text{H}_1.$$

H_1 is called the equivalent head of water, having pressure p and velocity v.

The following are numerical examples of the above principles:—

1. A sharp-edged circular orifice is 3 square inches in area, $C_v = {\cdot}97$, $C_c = {\cdot}64$. Find the quantity of water discharged through this orifice per hour, the head being 45 ft.

Let Q = cubic feet of water per hour,

$$= \frac{\text{A}}{144} \times C_v \times C_c \times v \times 3600,$$

$$= \frac{3}{144} \times {\cdot}97 \times {\cdot}64 \times 8\sqrt{45} \times 3600 = 2500,$$

in round numbers.

2. A rectangular notch is 10 ft. broad, and the head of water above the bottom of the notch is 5 ft. The whole weir is 40 ft. broad, and the total fall of the water is 15 ft. Find the number of cubic feet per second, and the available H.P. of the stream.

$$\text{Q} = 5{\cdot}35\, c\, b\, h\sqrt{h} = \text{cubic feet per second},$$
$$= 5{\cdot}35 \times {\cdot}595 \times 10 \times 5\sqrt{5},$$
$$= 355.$$

$$\text{H.P.} = 62{\cdot}5\ \text{Q} \times \text{fall in feet} \div 550,$$

since 550 foot-pounds = 1 H.P. per second,

$$= 62{\cdot}5 \times 355 \times 15 \div 550 = 605, \text{ nearly.}$$

3. Water is supplied by a scoop to a locomotive tender at a height of 7 ft. above the trough. Assuming that half the head is lost by friction, what will be the velocity of delivery when the train is running at 40 miles per hour, and what will be the lowest speed at which the operation is possible?

We must consider the motion of the water in the trough relative to the tender—

$$40 \text{ miles per hour} = 58\tfrac{2}{3} \text{ ft. per second.}$$

The head equivalent to this velocity

$$= \frac{v^2}{2g} = \frac{(58\frac{2}{3})^2}{64};$$

and as half of this is lost by friction, the effective head is

$$\frac{(58\frac{2}{3})^2}{128} = \text{H} = 26{\cdot}9.$$

Let V = velocity of delivery at a height of 7 ft. above the trough—

$$\frac{V^2}{2g} = H - 7;$$

$$V^2 = 64 \{26 \cdot 9 - 7\} = 64 \times 19 \cdot 9;$$

$$\therefore V = 8\sqrt{19 \cdot 9} = 35 \cdot 6.$$

The speed at which delivery ceases is such that;

$$\frac{v^2}{2g} = 7,$$

so that a column of water 7 ft. high would be maintained;

$$\therefore v = 8\sqrt{7} \text{ per second} = 14\tfrac{1}{2} \text{ miles per hour nearly.}$$

This arrangement is used to enable an express to pick up water while in motion. It is shown in fig. 9. The mouth of the scoop slices off a mass of water, and the relative motion of the trough to the train enables the water to overcome the head of 7 ft. The experiment made previous to

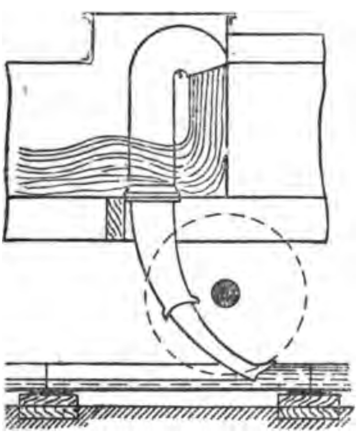

FIG 9

constructing this apparatus was as follows: A stream of water was allowed to issue from a water main at the speed of about 15 miles an hour, and a long $\frac{3}{8}$ in. pipe was bent at the bottom so as to face the current, and it was found that a stream was maintained through the pipe until its top was

raised $7\frac{1}{2}$ ft. above the level of the water. A stream of 15 miles an hour could theoretically be maintained by a head of $7\frac{1}{2}$ ft.; hence the water could rise no higher.

4. A direct-acting lift has a ram 9 in. diameter, and works under a constant head of $49\frac{1}{4}$ ft., allowing for weight of ram and friction of mechanism. Find the steady speed when lifting a load of 1,350 lb., and also the load raised at double that speed, neglecting friction of piping.

Let p = pressure per square foot,
W = load in pounds,
A = area of ram in square feet,

$$p = \frac{W}{A} = \frac{1350 \times 144 \times 4}{\pi \times 81} = 3060, \text{ nearly.}$$

Let v = speed in feet per second.

$$\frac{v^2}{2g} + \frac{p}{62{\cdot}5} = 49\tfrac{1}{4}$$

$$v^2 = 64\left\{49\tfrac{1}{4} - \frac{3060}{62{\cdot}5}\right\} = 64 \times \tfrac{1}{4}$$

$$v = 8 \times \tfrac{1}{2} = 4.$$

If the velocity were doubled,

$$\frac{v_1{}^2}{64} = 1.$$

$$\frac{p_1}{62{\cdot}5} + \frac{v_1{}^2}{2g} = 49\tfrac{1}{4}$$

$$\frac{p_1}{62{\cdot}5} = 48\tfrac{1}{4}$$

$$W_1 = p_1 A = \frac{62{\cdot}5 \times 48\frac{1}{4} \times \frac{\pi}{4} \times 81}{144}$$
$$= 1330 \text{ lb.}$$

A direct-acting lift is shown in fig. 10.

5. What is the pressure per square foot of area of the piston of a hydraulic engine, if the length of crank is 1 ft., and the number of revolutions 100 per minute, the piston being at the middle of its stroke, and the obliquity of the connecting rod neglected? The pressure produced by the accumulator is 750 lb. per square inch, friction of supply pipe neglected.

Here the head $H_1 = \dfrac{750 \times 144}{62{\cdot}5} = 1730$, nearly.

Let p = pressure required—

$$\frac{p}{62 \cdot 5} + \frac{v^2}{2g} = H_1$$

$$\frac{p}{62 \cdot 5} = H_1 - \frac{v^2}{2g}$$

$$v = \frac{2 \pi r N}{60}$$

where r = crank radius = 1 ft., and N = revolutions per minute = 100.

$$\therefore \frac{v^2}{2g} = \left(\frac{2 \times \pi \times 1 \times 100}{60}\right)^2 \left(\frac{1}{2 \times 32}\right) = 171.$$

$$\begin{aligned} p &= 62 \cdot 5\ (1730 - 171) = 62 \cdot 5 \times 1559, \\ &= 97{,}500 \text{ lb. per square foot}, \\ &= 676 \text{ lb. per square inch.} \end{aligned}$$

In the above examples the slide rule has been used for all calculations.

In the last example it was stated that the lift worked under a constant head. How this is obtained is shown in figs. 10 and 11, Mr. Ellington's lift.

The hydraulic ram which lifts the cage above is smaller in diameter than usual, and its size is determined by the strength required to carry the load, and not by the working pressure of water available. The lift cylinder A is connected to a cylinder B, beneath which is a cylinder C of larger diameter. There is a piston in each, connected by a ram D, fig. 11. Given a certain pressure of water, B is made of such a size that its full area enables the pressure above it to balance within a very little the weight of ram and cage when B is at the top and the ram is at the bottom of its stroke. The annular area E, fig. 11, is sufficient to overcome friction and lift the net load, while the whole pressures of E and B are communicated to the annular area J J in hydraulic connection with the lift cylinder, the area J J multiplied by the length of stroke giving a volume equal to that displaced by the ram. With proper proportions the force on the ram is the same at every point of the stroke, because, roughly speaking, when the ram is at the top of its stroke B and C are at the bottoms of their cylinders, and consequently the pressure per square inch on each of them, and therefore on the area J J, will be increased to such an extent as to exactly compensate for the loss of pressure caused by loss of head at the base of the ram.

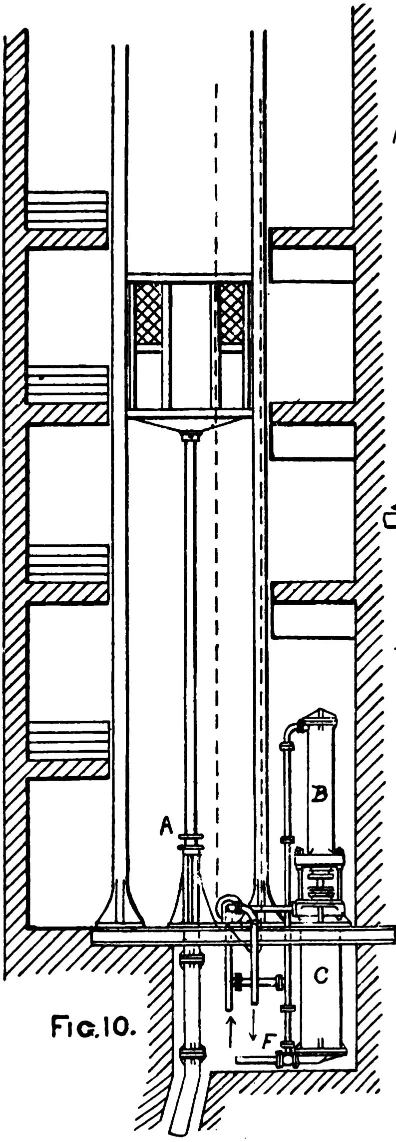

FIG. 10.

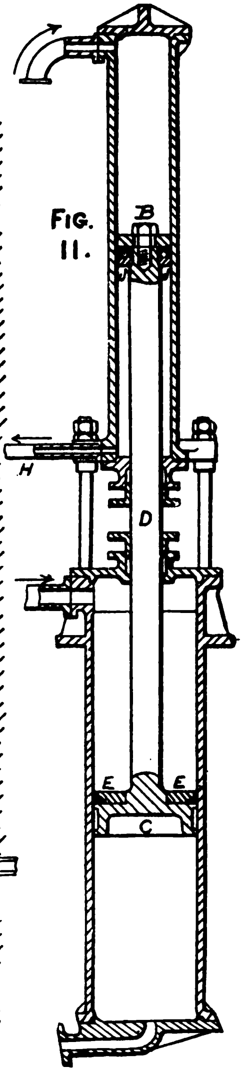

FIG. 11.

The mode of action of the lift is as follows: When the lift has to rise, the rope shown by a dotted line passing round two pulleys and up through the lift is pulled by the attendant; this admits water under pressure to C, while B is always in communication with the accumulator. Water is then forced out from the annular area J J into the lift cylinder, and the ram rises. When the lift descends, the valve is moved by the rope so as to allow the water from C to exhaust, as shown by the descending arrow, when the weight of lift and ram is sufficient to overcome the pressure on B, so that B rises as the lift descends, but the water in B is not wasted. To make good any leakage, the cock F, which is generally open to the atmosphere, can allow water to flow in under C, when it will raise it while the cage is at the bottom. This relieves the pressure in J J, and allows water from B to flow past the packing leather of that piston and replenish the space J. The power and economy of the lift can be increased when goods are being lowered by closing the cock F, so that a vacuum will be created beneath C, producing power enough to raise the empty lift without the expenditure of any water at all. The above description, with drawing, are taken from the Minutes of the Proceedings of the Institution of Mechanical Engineers, January, 1882.

The following example of such a lift is given there. Diameter of ram $3\frac{1}{2}$ in., diameter of B 11 in., diameter of C $21\frac{3}{4}$ in., available pressure $33\frac{1}{2}$ in., stroke of ram $50\frac{1}{2}$ ft., stroke of B C $8\frac{1}{6}$ ft. Useful load lifted 8 cwt. Since the water delivered from below J J = that supplied to the lift cylinder, the area of J J

$$= \frac{50\frac{1}{2} \times \cdot 7854 \times 3\frac{1}{2}^2}{8\frac{1}{6}} = 59\tfrac{1}{2} \text{ square inches.}$$

Now, let us suppose that B has to balance 515 lb. of the weight of the cage and ram when B is at the bottom and the ram is at the top of the lift cylinder; then, neglecting the small distance the bottom of the ram will be below the top of B, and supposing the pressure on B = $33\frac{1}{2}$ lb. per square inch, then, if A = area of B in square inches,

$$\frac{33\frac{1}{2} \times A}{59\frac{1}{2}} = p^1 = \text{pressure on J J per square inch,}$$

$$= \text{pressure on ram per square inch,}$$

$$= \frac{515}{9 \cdot 621},$$

$$A = 95 \text{ square inches, very nearly;}$$

i.e., diameter of B is 11 in., as given above.

The additional load, balanced by pressure on E E at the bottom of its stroke, is obtained as follows: It will be seen from the drawing that E is $10\frac{1}{2}$ ft. below B; ∴ pressure on E per square inch

$$= p_2 = 33\tfrac{1}{2} + \frac{10\frac{1}{2} \times 62{\cdot}5}{144} = 38 \text{ lb., very nearly;}$$

also the area of E E = $\cdot 7854 \times (21\frac{3}{4})^2$ − area of section of rod = 336 square inches; ∴ total pressure per square inch on J J, and ∴ on the ram,

$$= \frac{33\frac{1}{2} \times 95 + 38 \times 336}{59{\cdot}5}$$

$$= 268 \text{ lb. per square inch.}$$

Now, we shall show that this will be unaltered when the ram is at the bottom of the lift cylinder. Since B and C rise $8\frac{1}{6}$ ft., the pressure per square inch on each changes to 30 and 34·5 respectively. This causes on J J a pressure per square inch

$$= \frac{30 \times 95 + 34{\cdot}5 \times 336}{59{\cdot}5} = 242{\cdot}6\,;$$

but to this is added the pressure per square inch due to a head of $58\frac{2}{3}$ ft. = 25·4 lb. on the ram, because $58\frac{2}{3}$ is the distance the end of the ram is below J J; ∴ total pressure on ram = 268 lb. per square inch, as before.

The efficiency is not very high, if we consider that the efficiency

$$= \frac{\text{useful work done}}{\text{total work done}}$$

$$= \frac{8 \text{ cwt.} \times 50\frac{1}{2}}{\text{mean pressure on E} \times \text{area of E} \times 8\frac{1}{6}}$$

$$= \frac{8 \times 112 \times 50\frac{1}{2}}{36{\cdot}25 \times 336 \times 8\frac{1}{6}} = \cdot 455.$$

CHAPTER IV.

FRICTION OF PIPING.

SUPPOSE we have a sharp-edged thin flat plate, completely immersed in water, the relative velocity of water to plate being V. Let F be the force required to maintain the flow, or, in other words, to overcome the friction between the water and the surface of the plate; then

$$F = m A \frac{v^2}{2g}$$

where v is the velocity in feet per second, A is the area in square feet, and m is a coefficient depending on the surface of the plate. Experiment thus shows us that the friction is independent of the pressure of the water. The most important application of this law is to the friction of pipes. Fig. 12 represents a pipe in which are two pistons, A, B, at a distance apart $= l$ feet, and moving with a velocity v, the diameter of the pipe being d feet. Then, $A = \pi d l$;

$$\therefore F = m \pi d l \frac{v^2}{2g}.$$

This is equivalent to a pressure per square foot of section of pipe $= p$, where

$$p = \frac{m \pi d l v^2}{\frac{\pi}{4} d^2 2g} = m \frac{4l}{d} \frac{v^2}{2g}.$$

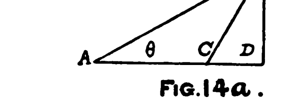

FIG. 12.

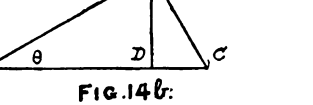
FIG. 13.

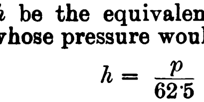

FIG. 14a.

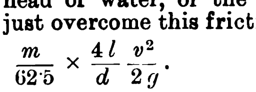

FIG. 14b.

Let h be the equivalent head of water, or the head of water whose pressure would just overcome this friction—

$$h = \frac{p}{62 \cdot 5} = \frac{m}{62 \cdot 5} \times \frac{4l}{d} \frac{v^2}{2g}.$$

But when water flows through a pipe, the whole stream does not flow with the same velocity as in the above case; consequently we are again indebted to experiment for a practical formula.

Darcy's formula—

$$h = Z\left\{1 + \frac{1}{12\,d}\right\} \times \frac{4\,l}{d} \times \frac{v^2}{2\,g},$$

which gives

$$F = 62{\cdot}5\,Z\left\{1 \times \frac{1}{12\,d}\right\} \times \pi\,l\,d \times \frac{v^2}{2\,g},$$

where $Z = {\cdot}005$ for new cast-iron pipes,
$Z = {\cdot}01$ for old incrusted cast-iron pipes.

It should be remembered that l and d are in feet, and v must be more than 4 in. per second. The above formula was obtained from a large number of experiments, and is very reliable, and we shall make use of it in subsequent examples, merely mentioning here two other formulæ.

Box's formula ("Practical Hydraulics")—

$$h = \frac{G^2 \times L}{(3\,d)^5},$$

where d is in inches, L in yards, G in gallons per minute.

Professor Unwin's formula—

$$h = \frac{m\,l}{d^x} \times \frac{v^n}{2\,g},$$

where l and d are in feet, and m, n, and x are constants depending on the roughness of surface of the pipe. For further particulars upon this equation the reader is referred to Professor Unwin's "Elements of Machine Design," Part II., section 5.

Example 1.—What is the force of friction in a pipe 3 ft. diameter, through which water is flowing at the rate of 4 ft. per second? The pipe is one mile long. Find also the head lost in friction, and the horse power required to overcome the resistance of the pipe, which is old and incrusted.

$$\begin{aligned} F &= 62{\cdot}5 \times Z\,A\,\frac{v^2}{2\,g}\left(1 + \frac{1}{12\,d}\right) \\ &= 62{\cdot}5\,Z\left(1 + \frac{1}{12\,d}\right) \times \pi\,d\,l \times \frac{v^2}{2\,g} \\ &= 8000 \text{ lb., putting } l = 5280,\ v = 4,\ Z = {\cdot}01,\ d = 3; \end{aligned}$$

also

$$\begin{aligned} h &= {\cdot}01\left(1 + \frac{1}{12\,d}\right) \times \frac{4\,l}{d} \times \frac{v^2}{2\,g} \\ &= 18{\cdot}1 \text{ ft.} \end{aligned}$$

To find the additional horse power of the pumps, we may either write

$$\text{H.P.} = \frac{\text{F} \times \text{feet per minute}}{33000} = \frac{\text{lbs.} \times \text{feet}}{33000}$$

$$= \frac{8000 \times 4 \times 60}{33000} = 58{\cdot}1\,;$$

or,

$$\text{H.P.} = \frac{h \times \text{lbs. per minute}}{33000}$$

$$= \frac{h \times v \times 60 \times \frac{\pi}{4} d^2 \times 62{\cdot}5}{33000}$$

$$= \frac{18{\cdot}1 \times 4 \times 60 \times \frac{\pi}{4} \times 3^2 \times 62{\cdot}5}{33000}$$

$= 58{\cdot}1$, as before.

Example 2.—A pumping engine is required to deliver 240 cubic feet of water per minute, through a pipe three miles long, to a head of 100 ft. above the level of supply; diameter of pipe, 18 in. Find the I.H.P. of the pumping engine, assuming that work done on water $\div$ indicated work of steam $= {\cdot}\dot{6} = \frac{2}{3}$.

$$h = \text{Z}\left\{1 + \frac{1}{12\,d}\right\} + \frac{4\,l}{d}\,\frac{v^2}{2\,g},$$

and

$$v = \frac{240}{60} \times \frac{1}{\frac{\pi}{4} d^2} = 2{\cdot}26, \text{ if } d = 1\tfrac{1}{2}\text{ ft.}$$

$$\therefore h = {\cdot}01\left\{1 + \frac{1}{18}\right\} \times \frac{12 \times 5280}{1\frac{1}{2}} \times \frac{(2{\cdot}26)^2}{64} = 35{\cdot}6.$$

$$\therefore \text{Total head} = 100 + 35{\cdot}6 = 135{\cdot}6.$$

$\therefore$ I.H.P. of pumping engine $\times$ efficiency

$$= \frac{\text{head} \times \text{lbs. per minute}}{33000}$$

$$\therefore \text{I.H.P.} = \frac{135{\cdot}6 \times 240 \times 62{\cdot}5}{33000 \times {\cdot}\dot{6}} = 92{\cdot}5.$$

The above presented itself to candidates who took the second part of the Paper on Machine Construction (Hons.) for 1890, in the design of the pumping engine, and is a very practical question.

CHAPTER V.

Losses of Energy from Sudden Changes of Velocity and Direction.

Whenever water flowing in a passage suddenly changes its direction or velocity, there is a loss of energy. In fig. 13 the water is flowing from a pipe of small diameter into one of larger diameter, the change of section being sudden. Let A, A_1 be the sections of the smaller and larger pipes in square feet, and v, v_1 the corresponding velocities in feet per second, then

$$Q = A v = A_1 v_1$$

where Q = cubic feet per second.

Consider a mass of water contained between two planes, which we shall call $C D$ and $E F$, the former cutting the smaller pipe and the latter the larger, both being perpendicular to its axis, and suppose in a small period of time t that this mass changes its position so that it is bounded by two planes $C^1 D^1$ and $E^1 F^1$, then the loss of momentum is the difference of the momenta of $C C^1 D^1 D$, and $E E^1 F^1 F$. Let W be the weight of each of these—

$$W = 62{\cdot}5\, Q t = 62{\cdot}5\, A_1 v_1 t.$$

Change of momentum

$$= \frac{62{\cdot}5 \,.\, A_1 v_1 (v - v_1) t}{g} =$$

impulse of all the forces acting on the water parallel to the axis. Let p be the pressure per square foot in the smaller pipe, p_1 that in the larger, and let p_o be the pressure on the annular space where the pipes meet, then

$$\{p_1 A_1 - p A - p_o (A_1 - A)\} t = \frac{62{\cdot}5\, A_1 v_1 (v - v_1) t}{g};$$

but p_o has been shown by experiment to be equal to p_1, hence

$$\frac{(p_1 - p) A_1}{62{\cdot}5} = \frac{A_1 (v v_1 - v_1^2)}{g};$$

$$\frac{p_1 - p}{62{\cdot}5} = \text{gain of pressure head}$$

$$= \frac{v v_1 - v_1^2}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let H = head due to pressure and velocity in the first pipe, H_1 = that in the second

$$H = \frac{p}{62{\cdot}5} + \frac{v^2}{2g}$$

$$H_1 = \frac{p_1}{62{\cdot}5} + \frac{v_1^2}{2g}$$

$$\begin{aligned} H - H_1 &= \text{loss of head} \\ &= \frac{p - p_1}{62{\cdot}5} + \frac{v^2 - v_1^2}{2g} \\ &= \frac{2v_1^2 - 2vv_1 + v^2 - v_1^2}{2g} \\ &= \frac{(v - v_1)^2}{2g} \quad \text{. . . . . (2)} \end{aligned}$$

This is, therefore, the loss of energy per pound of water.

Energy will also be lost by a sudden change of direction. In figs. 14*a*, 14*b* (page 15), A B is the velocity v, which is suddenly changed to v_1 = A C, the change of direction being through the angle B A C = θ.

Then the loss of head or energy per pound

$$\begin{aligned} &= \frac{BC^2}{2g} = \frac{1}{2g}(v^2 + v_1^2 - 2vv_1 \cos\theta) \\ &= \frac{1}{2g}(v^2 + v_1^2 - 2v_1 AD) \quad \text{. . (3)} \end{aligned}$$

where B D is perpendicular to A C.

Now, for a given change of direction B C will clearly be a minimum when B C = B D; that is, when v_1 = A D = $v \cos\theta$, an important result, as will be seen later on. Under certain circumstances it may be preferable to obtain by the change of direction and velocity a maximum gain of pressure; as before, let H = the equivalent head before change of direction, and H_1 the same after change of direction.

$$H = \frac{p}{62{\cdot}5} + \frac{v^2}{2g}$$

$$H_1 = \frac{p_1}{62{\cdot}5} + \frac{v_1^2}{2g}$$

and $$H - H_1 = \frac{BC^2}{2g}$$

$$\therefore \frac{p_1 - p}{62{\cdot}5} = \frac{v_1 v \cos\theta - v_1^2}{g} \quad \text{. . . (4)}$$

= the gain of pressure head.

We may also write

$$\frac{p_1 - p}{62{\cdot}5} = \frac{v^2 - v_1{}^2}{2g} - \frac{BC^2}{2g}$$

$$= \frac{AB^2 - AC^2 - BC^2}{2g}$$

$$= \frac{AC.CD}{g} \text{ when } AC < AD \text{ (fig. 14}a\text{)}$$

$$= -\frac{AC.CD}{g} \text{ when } AC > AD \text{ (fig. 14}b\text{).}$$

so that the gain may be found graphically.

Now, for a maximum value of $p_1 - p$, A C. C D must be a maximum and A C < A D. Now, it is a well-known fact that if a straight line of fixed length, such as A D, which is constant when v and θ are fixed, be divided into two parts such as A C, C D, that the greatest product A C. C D is obtained when $AC = CD = \frac{1}{2} AD$. $\therefore p_1 - p$ is a maximum when $v_1 = \frac{1}{2} v \cos\theta$, and

$$\frac{p_1 - p}{62{\cdot}5} = \frac{1}{4}\frac{v^2 \cos^2\theta}{g}$$

Thus we may place these two results side by side—

For a minimum loss of energy $v_1 = AD = v\cos\theta$ . . (5)

For a maximum gain of press. head $v_1 = \frac{1}{2} AD = \frac{1}{2} v\cos\theta$. (6)

Example 1.—Find the energy in foot-pounds lost per second if 1,000 cubic feet of water per minute flows through a pipe 20 in. diameter suddenly enlarged to 30 in. diameter.

$$\text{Loss of energy per pound} = \frac{(v - v_1)^2}{2g} = h, \text{ say}$$

$$v = \frac{1000}{60} \times \frac{144}{{\cdot}7854 \times 20^2} = 7{\cdot}65$$

$$v_1 = \frac{1000}{60} \times \frac{144}{{\cdot}7854 \times 30^2} = 3{\cdot}4$$

$$\therefore h = \frac{(7{\cdot}65 - 3{\cdot}4)^2}{64} = {\cdot}282$$

$$\therefore \text{energy lost per second} = \frac{{\cdot}282 \times 1000 \times 62{\cdot}5}{60} = 294 \text{ ft.-lbs.}$$

Example 2.—Water is flowing with a velocity of 8 ft. per second in a pipe 2 square feet section, the total head causing the flow being 81 ft.; the pipe is suddenly enlarged to 4 square feet section. Find the pressure per square foot in the second pipe and the loss of head.

This last $= h = \dfrac{(v - v_1)^2}{2g}$, and $v_1 = \frac{1}{2} v = 4$.

$$\therefore h = \frac{4^2}{64} = \tfrac{1}{4}.$$

Hence,

$$\frac{p_1}{62 \cdot 5} + \frac{v_1{}^2}{2g} = \frac{p}{62 \cdot 5} + \frac{v^2}{2g} - h = \mathrm{H} - h = 81 - \tfrac{1}{4} = 80\tfrac{3}{4}.$$

$$\therefore \frac{p_1}{62 \cdot 5} = 80\tfrac{3}{4} - \frac{v_1{}^2}{2g} = 80 \cdot 5, \therefore p_1 = 5031 \text{ lb.}$$

Example 3.—A stream flowing at 16 ft. per second has its direction suddenly altered through an angle of 30 deg., the new velocity being 8 ft. per second. Find graphically or by calculation the loss of energy per pound, and the pressure of the water, if originally it was 20 lb. per square inch.

For the graphic method take fig. 14*a*. Make AB = 16, BAC = 30 deg., AC = 8. Then $\dfrac{\mathrm{BC}^2}{2g}$ is the loss of head, and

$$\frac{p_1 - p}{62 \cdot 5} = \frac{\mathrm{AC.CD}}{g}$$

and $$p = 20 \times 144.$$

$\therefore$ if P_1 and P are the pressures per square inch,

$$\mathrm{P}_1 = \mathrm{P} + \frac{62 \cdot 5\ \mathrm{AC.CD}}{g}$$

$$= 20 \cdot 63 \text{ per square inch.}$$

By calculation

$$\frac{\mathrm{BC}^2}{2g} = \frac{v^2 + v_1{}^2 - 2 v v_1 \cos\theta}{2g}$$

$$= \frac{16^2 + 8^2 - 2 \times 16 \times 8 \times \frac{\sqrt{3}}{2}}{64}$$

$$= 1 \cdot 54 \text{ ft.}$$

$$\frac{p_1 - p}{62 \cdot 5} = \frac{v_1 v \cos\theta - v_1{}^2}{g}$$

$$\mathrm{P}_1 = \mathrm{P} + \frac{62 \cdot 5}{144} \; \frac{16 \times 8 \times \frac{\sqrt{3}}{2} - 8^2}{32}$$

$$= 20 \cdot 63 \text{ lb. per square inch.}$$

CHAPTER VI.

Hydraulic Engines.

We have always found that theoretical principles, although absolutely necessary, are best administered in small doses; we shall therefore turn from them to describe a few types of hydraulic engines for producing rotation. First—and, we believe, foremost—comes the Brotherhood three-cylinder single-acting engine (figs. 15, 16, 17, 18). There are three cylinders A, always open at their inner ends, attached to a central chamber B, and a single crank pin receives the

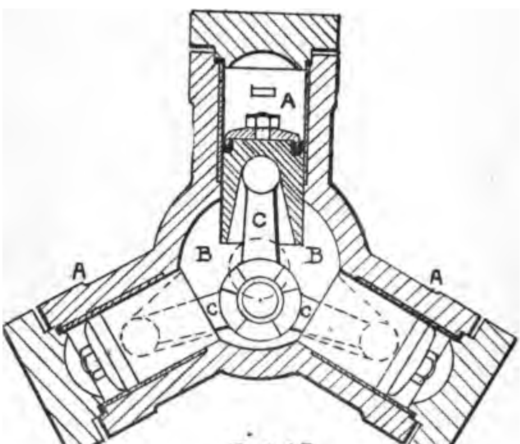

Fig 15.

pressure on the three pistons, acting through the struts C. The water is admitted and exhausted by means of a single valve V, made of hard phosphor bronze, which is shown in section in fig. 16, and in end elevation in fig. 17. It revolves in the valve chest E, being driven from the main crank by the crank F, the square end of whose shaft fits into the valve. The valve chest E is bolted to the cover G, in which are three passages leading from E to the passages in the main casting, which open into the three cylinders. The valve is shown exhausting from the upper cylinder by

means of H to the exhaust pipe X. In half a revolution it will admit water under pressure to the same cylinder. The balance ring on the valve will be noticed, the pressure on the face of the valve being thus reduced to 300 lb. per square inch. As there is no dead centre, the engine will start in

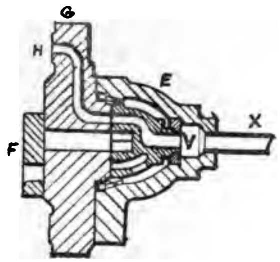

Fig 16.

Fig 17.

any position of the crank pin, and no flywheel is required. Its most useful application is to a capstan, when the shaft is placed vertically, the engine being beneath the capstan. The whole arrangement of capstan and engine can be rotated on a horizontal axis, so as to bring the engine above in

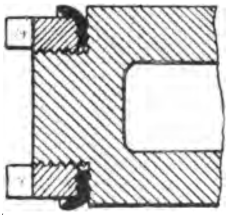

Fig 18.

order to examine it, or for the execution of repairs. In fig. 18 another and better method of packing the piston is shown; bucket leathers are used, of shallower section, so as to be stiffer in the lip, and have the flesh side of the hide outside. Figs. 15—17 are one-twelfth full size.

Figs. 19, 20, and 21 show Schmid's hydraulic motor, designed for the use of small manufacturers and for domestic purposes. It is an oscillating engine, the cylinder resting

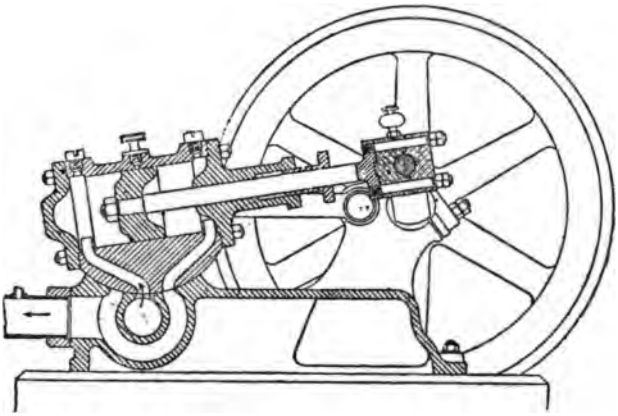

FIG. 19.

on a cylindrical surface, about the axis of which the oscillation takes place. The bedplate has a corresponding hollow

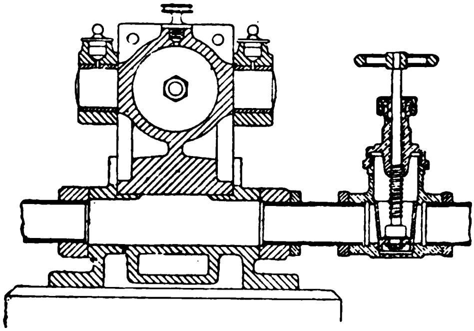

FIG. 20.

containing the admission and two exhaust passages, the motion of the cylinder opening these at the proper time.

Thus (fig. 19) the piston is moving to the right, water under pressure is entering (as shown by the arrow) from the central port, while the exhaust is taking place through the right port. On either side of the cylinder are short gudgeons or pins, whose axis is the axis of oscillation. These have their bearings in two levers (figs. 19, 21), pivoted at one end to the frame, near the crank axle. By means of a hand wheel and screw, acting on the other end of each lever, the cylinder can be pressed down upon its bearing surface; and by

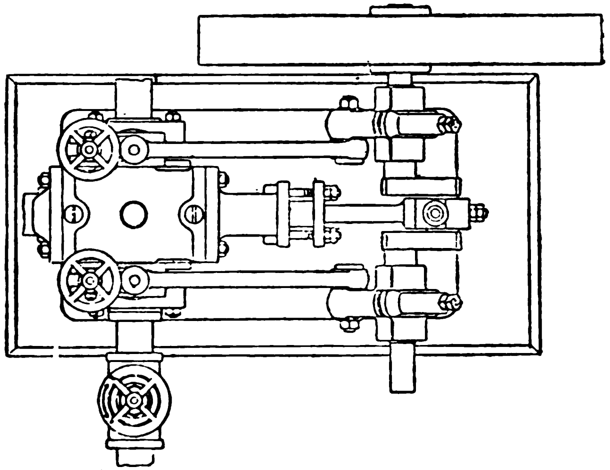

FIG. 21.

unscrewing the hand wheel the face of the cylinder can be lubricated or inspected, as the cylinder can then be lifted quite off the bed. The piston is solid, a leather collar being used in large machines, while in small ones it is turned and ground in. A small copper air vessel, from two to two and a half times the cylinder capacity, is used in connection with the engine, and for larger sizes a small air pump, to replace loss by leakage and absorption of air by the water. The necessity of air vessels or neighbouring accumulators will be shown when the theory of the hydraulic motor is dealt with, but the reader will readily see that the varying speed of the piston will not merely require the acceleration and retardation of the moving parts at different points of

the revolution, but also of the water in the cylinder and supply pipes, which, if there be no air vessel or accumulator near the engine, will at one moment decrease and at another increase the effective pressure, causing at high speeds dangerous shocks, which it is the duty of the designer to avoid. We shall, however, treat of this more fully later on. The maker of these engines guarantees an efficiency of 80 per cent, and experiment shows that it rises as high as 90 per cent.

CHAPTER VII.

Theory of the Hydraulic Engine.

If an engine is moving very slowly, the pressure on the piston is constant and equal to that of the accumulator; but when it is moving at any speed, the inertia of the reciprocating parts, and of the water in the pipes leading from the accumulator or air vessel, and also the friction of the water in the pipes, modify considerably the pressure at each point of the stroke. All these evils can be practically eliminated by having a sufficiently large air vessel, or an accumulator near enough to the engine, because either of these modifies the variation of velocity of flow in the pipes, and thus reduces the friction, and practically eliminates the change of pressure caused by the acceleration and retardation of an otherwise long column of water. We say "reduces the friction," because, as the friction varies as the square of the velocity at any instant, it will be less if the velocity be constant than if it varies, as it would do if its velocity was proportional to that of the piston. If there were no air vessel or accumulator near the engine, a long column of water in the supply pipe would have a varying velocity, equal to that of the piston multiplied by the ratio of the area of cylinder to area of pipe—*i.e.*, if there is only one cylinder. With two cylinders and cranks at right angles, the reader will readily see that the variation of flow would be less; and with three, as in the Brotherhood engine, there will be hardly any variation at all, except in the cylinders and their passages, and therefore an air vessel is not needed. To see this graphically, figs. 22, 23, 24 show the velocities of flow in the supply pipes of a double-acting single-cylinder engine, a double-acting two-cylinder engine with cranks at right angles, and a three-cylinder single or double-acting engine with cranks at 120 deg. In the

figures the ordinates represent velocities of flow, and the base line a revolution of the crank pin ; the horizontal line above and parallel to the base line shows the mean velocity of flow. In these three cases, taking 100 as the mean velocity, 157 represents the maximum in the first case, 111

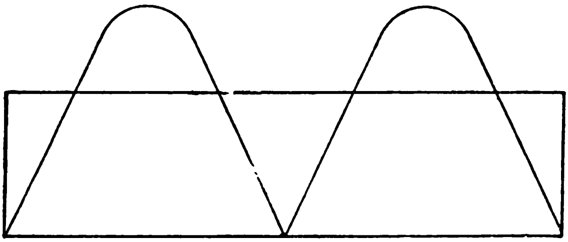

Fig 22.

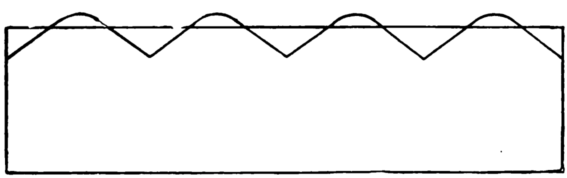

Fig 23.

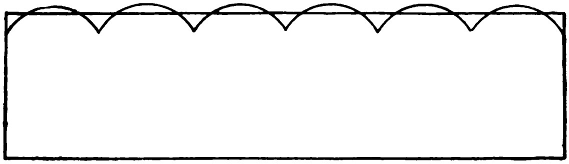

Fig 24.

in the second, and 104·7 in the third ; while the minima are 0, 78·79, and 90·69 respectively, showing variations of 157, 32·21, 14·01 per cent.

Let us now suppose that we have a single-cylinder double-acting engine without an air vessel, and let the coefficient

of hydraulic resistance referred to the velocity of the piston be F; that is to say, if the piston be moving with a velocity V, the head lost by hydraulic friction, &c., is $F \frac{V^2}{2g}$; then F is a constant quantity, because the frictional loss of head varies as the square of the velocity at each point, and that velocity must bear some fixed ratio to the piston velocity. Hence, if A B be the length of stroke, and a line M N be drawn equal to $(1 + F) \frac{V^2}{2g}$, and a similar construction be made at each point of the stroke, a curve A K B will be obtained, which will show graphically the reduction of

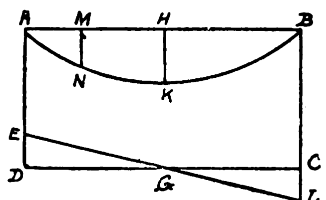

Fig 25.

pressure in feet of water on the piston caused by the velocity and friction of the water. F must be obtained by calculating the frictional losses caused by piping, &c., between the engine and the accumulator. The curve A K B is a parabola, which may be readily seen by anyone who knows the properties of this curve, and remembers that the velocity of the piston $V = v \sin \theta$ neglecting the obliquity of the connecting rod, v being the velocity of the crank pin, and θ the angle between the line of stroke and the crank centre line. But this is not the only alteration of head, for the continually varying velocity of the piston makes it necessary for the water in the pipes to increase and reduce its velocity before and after mid-stroke respectively.

Let W = weight of piston and water in the cylinder,
W^1 = weight of accelerated column of water in the pipes,
a = area of pipe section in square feet,
A = area of cylinder in square feet,
$2r$ = length of stroke;

then the velocity of water in the pipe $= \frac{A}{a} \times$ velocity of water in cylinder. $\therefore$ the acceleration of the water in the pipe $= \frac{A}{a} \times$ acceleration of the water in the cylinder.

But this latter acceleration $= \frac{v^2}{r}$ at the beginning, and $-\frac{v^2}{r}$ at the end of the stroke, while at the middle it is zero. No energy is lost by this, as whatever pressure is lost at the beginning is restored at the end of the stroke. At the beginning of the stroke the force required to produce acceleration may be considered as made up of two parts: firstly, that required for the water in the pipe; secondly, that for the water in the cylinder and for the piston.

$$P_1 = \frac{W}{g}\frac{v^2}{r}$$

$$P_2 = \frac{W^1}{g}\frac{A}{a}\frac{v^2}{r}$$

Loss of head in the pipe

$$= \frac{P_2}{62{\cdot}5\,a} = \frac{W^1}{62{\cdot}5\,g\,a}\frac{A}{a}\frac{v^2}{r} = \frac{l}{g}\frac{A}{a}\frac{v^2}{r},$$

where l is the length of pipe.

Let $\frac{A}{a} = n$,

then the loss of head in the pipe is $\frac{n\,l}{g}\frac{v^2}{r}$.

The loss of head in the cylinder is $\frac{P_1}{62{\cdot}5\,A}$

$$= \frac{W}{62{\cdot}5\,A} \cdot \frac{v^2}{g\,r}$$

$$\therefore \text{ total loss of head} = \frac{v^2}{g\,r}\left\{n\,l + \frac{W}{62{\cdot}5\,A}\right\}$$

If D E and C L be drawn equal to this value in fig. 25, and E G L, a straight line, be drawn, then the ordinates of D E G L C will represent decrease and increase of head caused by acceleration, and the diagram showing effective pressures on the piston will be E A N K B L, E L being the base line. Now, with high pressures of water an accumulator, or air vessel charged with compressed air, will, if placed as

near the engine as possible, lessen these evils so that they are unimportant; with moderate pressures an air vessel without compressed air will be sufficient. If the curve AKB touched or cut the line EL, violent shocks would occur. If two or three cylinders are used, instead of one larger one using the same quantity of water, the work lost by friction in a supply pipe of given length will be less in consequence of the more uniform flow.

If we take as the unit the loss of work per revolution when the flow is uniform, then when the same quantity of water supplies three cylinders the loss is less than 1·01; if there are two cylinders, the loss is less than 1·03; while with one single cylinder it is 1·64, showing the advantage of using several cylinders if no air vessel or neighbouring accumulator is used. The greater uniformity of twisting moment is also another and more important advantage.

The above theory is not any help in designing a hydraulic engine, except in so far as it shows us the evils to be avoided, and the simple means by which they may be avoided. In addition to the loss caused by fluid friction, there are also losses due to sudden enlargements and contractions, sharp bends in passages and pipes, and energy wasted in the off-blowing water. The mean effective pressure per square foot when an air vessel is used is

$$p_1 = p - \frac{2}{3}(1 + \mathrm{F}) \cdot 62{\cdot}5 \cdot \frac{v^2}{2g} = p - 1{\cdot}64\,(1 + \mathrm{F}) \cdot 62{\cdot}5\,\frac{\mathrm{V}^2}{2g}$$

where V is the mean velocity of the piston, and v is the velocity of the crank pin, and F is a coefficient of resistance referred to the velocity of the piston, including all the above losses: p is the pressure per square foot due to the accumulator, less what must be subtracted for friction of supply pipes in which the velocity is now uniform.

The efficiency is therefore

$$\eta = \frac{p_1}{p} = \frac{p - 1{\cdot}64 \cdot 62{\cdot}5 \cdot (1 + \mathrm{F})\,\frac{\mathrm{V}^2}{2g}}{p}$$

The ordinary form of hydraulic engine is incapable of economic regulation. If the power required decreases, it is impossible to decrease the power of the engine. It therefore is necessary to have some special arrangement for altering the power of the engine. In hydraulic cranes the valves are arranged so that both sides of the piston can be placed in communication with the accumulator, and consequently the

effective area on which the water acts is only equal to the section of the rod, but this only allows one variation of power. Another method of variation is to permit the valve to cut off the supply of compressed water, the piston sucking in a supply of water from a tank while it continues the stroke. We do not know of any engine producing rotary

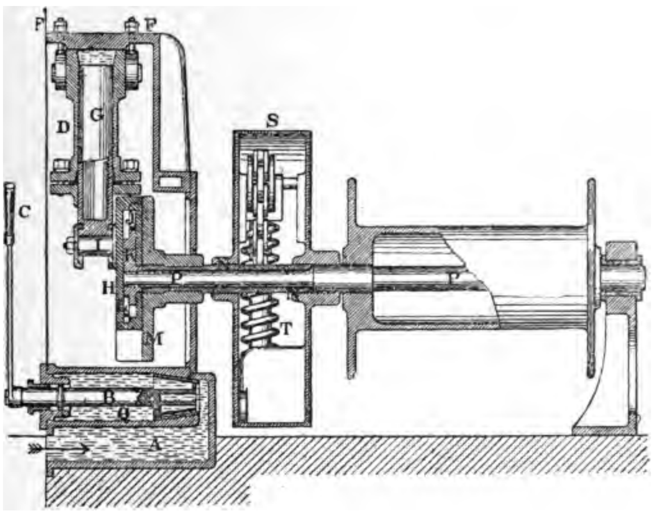

FIG. 26.

motion that varies the mean pressure in this way. The two remaining quantities capable of alteration are the number of revolutions and the length of stroke, and it is this latter that is varied in Mr. John Hastie's engine, described in the Proceedings of the Mechanical Engineers in 1879.

Figs. 26, 27, 28, 29, 30, 31 show the construction as applied to a hoist. There are three oscillating cylinders D, to which water is admitted by passages E, which act alternately as admission and exhaust ports; thus valves are not required. A is the inlet pipe, B a cock worked by the handle C, which controls the action of the hoist, and can be used as a reversing valve at the extreme positions, and as a brake in the central position. In the latter case both ports of the cylinders are placed in connection with Q, the exhaust passage, which is made with a bend, so as to contain at least

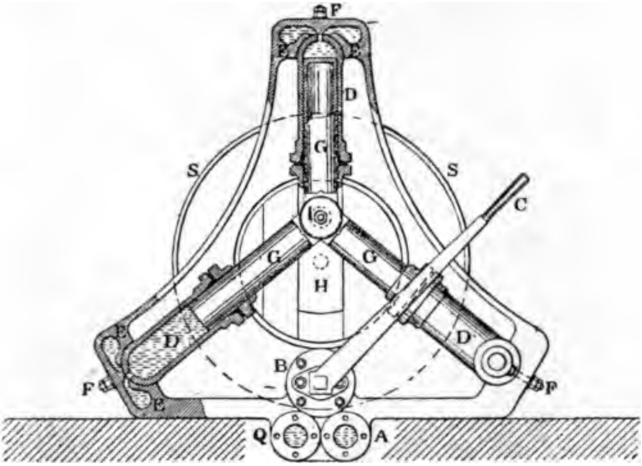

Fig. 27.

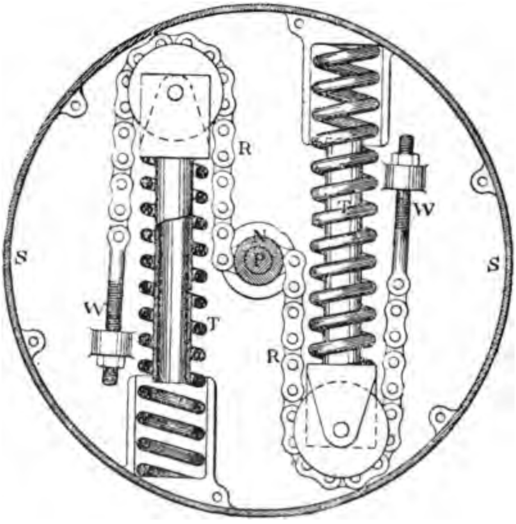

Fig. 28.

as much water as will fill three cylinders. The crank pin I is attached to a block H (figs. 26, 27, 29, 30), sliding in a radial groove in M, the crank disc. At the back of this block, revolving on pins projecting from it at either end, are two small rollers J and L, which run on the circumference of a peculiarly shaped cam K (figs. 26, 31). This cam is keyed to a spindle P, passing through a concentric hollow

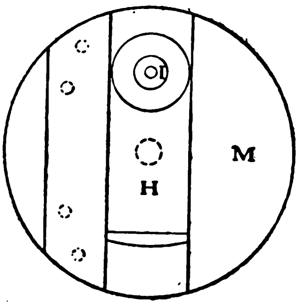

FIG. 29.

shaft N, carrying the crank disc. Supposing the spindle to be held fast while the hollow shaft and crank disc revolve, the block H will be displaced radially by the crank K, and with it the crank pin, thus altering the stroke of the pistons. A hollow cylindrical casing S is keyed to the spindle P, and

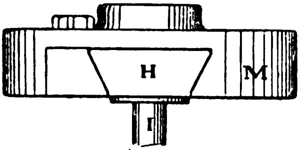

FIG. 30.

runs loose on N; within this are two rollers, carried in forks at the ends of hollow rods. These rods, with the rollers, are pressed outwards towards the circumference by helical springs T, while over each roller passes a chain R, one end of which is attached to a snug W, projecting from the side of the drum, the other to the hollow shaft N.

When water is admitted to the cylinder under pressure, the crank disc begins to revolve, and with it the hollow shaft

N, while the spindle P carrying the cam K is held fast by the resistance of the load, applied through the hoisting chain to the circumference of the chain barrel. The result of this is that the chains R are wound up on N; at the same time, owing to the motion of the crank disc relatively to the cam, the block H with the crank pin is pushed outwards, and the stroke is increased. The winding up of the chain compresses the springs, and this compression, and the simultaneous increase of the stroke, go on until the resistance of the springs balances that of the pulley, when the latter is driven round, and a state of equilibrium established, which lasts as long as no change occurs in the load or pressure. If the load is increased, a motion of the crank disc relatively to the cam again takes place, until the turning moment on the crank

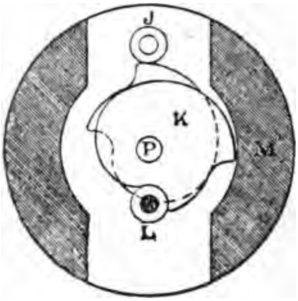

FIG. 31.

equals that of the load on the pulley by the alteration of the crank radius.

In engines working with very high pressures springs are not employed, but are replaced by two water plungers working in cylinders, which are in connection with a supply pipe through the centre of the shaft. The chains R (fig. 28) in this case are not fixed as in fig. 28, but are wound on cams, so that greater power is required to force back the rams in proportion as the chains R act at an increasing distance from the centre of the shaft. The following are the results of experiments made on a small hoist for the Greenock Infirmary. The lift was 22 ft., and the pressure per square inch 80 lb. = 11,520 per square foot.

Weight lifted,	chain alone—	427,	633,	745,	857,	969,	1081,	1193 (pounds).
Water used—	$7\frac{1}{2}$	10,	14,	16,	17,	20,	21,	22 (gallons).
Efficiencies—	0	·51,	·54,	·56,	·60,	·58,	·615,	·65

$$\text{The efficiency} = \frac{\text{useful work}}{\text{total work}} = \frac{\text{weight} \times \text{lift} \times 6{\cdot}25}{11{,}520 \times \text{gallons used}}$$

It is not the efficiency of the engine alone, but that of engine and hoist. The maximum efficiency of the engine would be about $\frac{\cdot 65}{\cdot 75} = \cdot 87$, nearly,

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CHAPTER VIII.

The Turbine.

A turbine is a form of water-wheel which makes use of the energy of water as it flows between curved vanes or channels, in which its course is so altered that it exerts a reactionary force, thereby causing and maintaining motion. This action, it will be readily seen, is an application of Newton's second and third laws of motion.

Before giving any description or classification of turbines, we shall first consider the action of water on curved vanes.

Let A B represent a vane moving in the direction indicated by the arrow, with uniform velocity c; let O A represent the direction and magnitude v of a thin stream of water entering at A in such a manner that there is no sudden change of velocity or direction of flow. In order that this may be so, the parallelogram $O\,v_1\,A\,c$ must be the parallelogram of velocity of the water, A C being the tangent to the vane at A, and $O\,v_1$, $O\,c$ must represent v_1, the velocity of the water relative to the vane, and c the velocity of the vane. If, after drawing A O, O c (fig. 33), c A were not a tangent to the vane, as A k at A, then the water would not glide along the vane without shock, but its direction and velocity would be suddenly altered at A. We must leave this latter case at present, and return to fig. 32. A perpendicular $A\,w_1$ upon O c gives us the tangential velocity, or velocity of whirl w_1 of the water at entry; the angles ϕ and α may also be noticed.

Again, at the point B, where the water leaves the vane, $B\,v_2 = v_2$ is the relative velocity, $B\,e = c$, $B\,u = u$ the total velocity, and $B\,w_2$ is the velocity of whirl at discharge, $B\,w_2\,u$ being a right angle, and the angles θ, β should be noted.

Let W be the weight in pounds of water passing in time t, and let P be the component, parallel to the direction of

motion of the vane, of the re-action between water and vane—

$$\frac{W}{g}(w_1 - w_2) = P\,t,$$

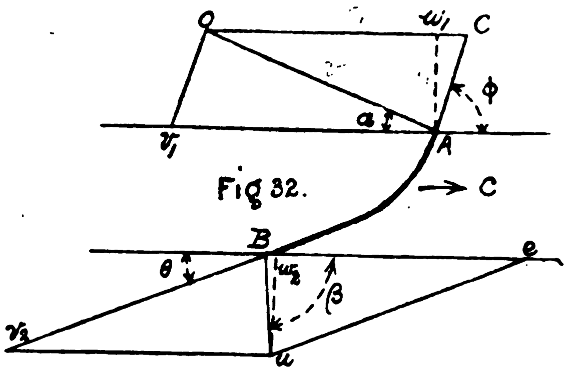


Fig 32.

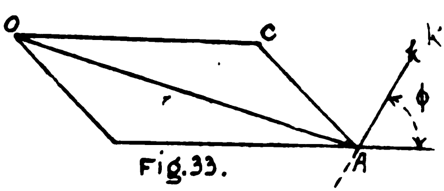


Fig.33.

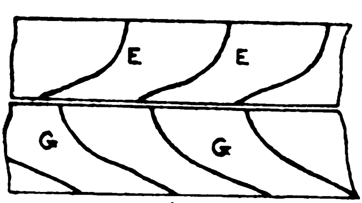


Fig 34.

because the momentum is changed from

$$\frac{W\,w}{g} \text{ to } \frac{W\,w_2}{g}$$

in time t by a force P. Hence the work done in time t on the vane

$$= c\,t \times \text{P foot-pounds};$$

$$= c \times \text{P}\,t = \frac{\text{W}\,c}{g}\,(w_1 - w_2),$$

a quantity independent of t.

The work done per pound

$$= \frac{c}{g}\,(w_1 - w_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If entry is to take place without shock, and certain velocities and angles are assumed, it is clear that by construction or trigonometry the remaining velocities and angles can be found. For example, if we assume β, θ, and v_2, then u and c are found by the parallelogram $\text{B}\,v_2\,u\,e$. Again, if $v = \text{OA}$ is known with α and c, the parallelogram $\text{A}\,c\,\text{O}\,v_1$ will enable us to find v_1 and ϕ.

However useful graphical methods may be in saving calculation, it will be found that very little more time is required, and greater accuracy is obtained, by the use of a slide rule, and a table of sines, cosines, tangents, &c. Also it is quite possible that calculations must be made when a drawing board is not obtainable, when the following equations will be useful:—

$$\left.\begin{aligned} w_1 &= v \cos \alpha \\ c - w_1 &= v_1 \cos \phi \\ c \sin \phi &= v \sin (\alpha + \phi) \\ v_1 \sin \phi &= v \sin \alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\left.\begin{aligned} w_2 &= u \cos \beta \\ c - w_2 &= v_2 \cos \theta \\ c \sin \theta &= u \sin (\beta + \theta) \\ v_2 \sin \theta &= u \sin \beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Supposing, as before, β, θ, and v_2 known, then the fourth equation of (9) gives us u, and the first will now give w_2, and c may be found from the second or third. Again, if v, α, and c are known, w_1 is found from the first of equations (8); ϕ may be calculated from the third, which may be written

$$c - w_1 = v \cot \phi \sin \alpha,$$

and v_1 may now be obtained from the second or fourth.

The equations (8) are unaltered if ϕ be greater than a right angle; and the above reasoning will apply equally well when the vane moves, not in a straight line, but in a circle whose plane is perpendicular to the paper, the axis of rotation being therefore parallel to the paper.

The reader will now be able to understand the axial or parallel flow turbine, through which the water flows in a direction parallel to the axis of rotation. Figs. 34 and 35

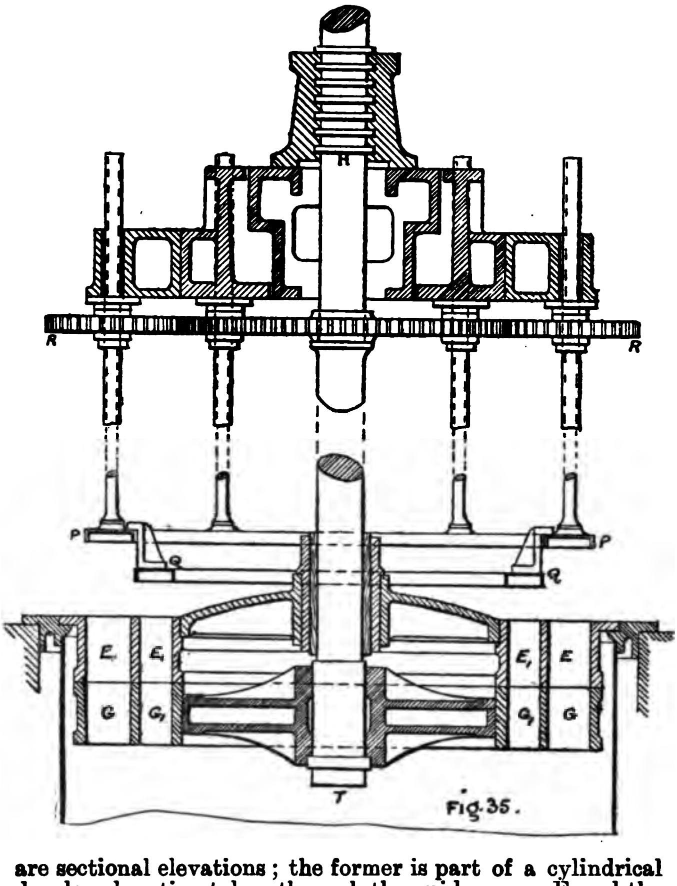


Fig. 35.

are sectional elevations; the former is part of a cylindrical developed section taken through the guide vanes E and the wheel vanes G, the direction of motion of G being to the left, while the guide vanes are fixed. The water, collected in a

reservoir above, flows downwards into the guide apparatus, in which it is given a forward velocity v; leaving these, it should enter the wheel G without shock, and flow out vertically after it has imparted some of its energy to the wheel in consequence of the change of velocity of whirl from w_1 to w_2, which latter should be zero, the reason for which will be afterwards explained. Fig. 35 is a vertical section through the axis of rotation. This drawing and description are taken from the Proceedings of the Institution of Mechanical Engineers, from Mr. Morrison's paper on "The Transmission of Power by Turbines and Wire Ropes." There are three such turbines at Schaffhausen, on the Upper Rhine, from which the necessary power is obtained.

Each turbine is $9\frac{1}{2}$ ft. outside diameter, and is carried by a vertical wrought-iron shaft 8 in. diameter. The fall of water varies from 12 ft. to 16 ft., according to the quantity in the river. Each turbine is capable of developing about 250 horse power when using about 200 cubic feet of water per second, with a fall of 12 ft. We do not know if very accurate tests of these wheels have been made, for the above figures would show an extraordinary efficiency of 91·6 per cent, whereas 85 per cent would be considered very good. During periods of flood the quantity of water can be increased to about 280 cubic feet. The average number of revolutions is 48 per minute, giving a circumferential speed of 24 ft. per second. The turbine wheel is in two parts, and is called a double axial turbine. When the water in the river is low there is a greater difference of level between the upper and lower water, so that the head is increased, during which period the outer rings E G of guide vanes and wheel are used, the inner rings E_1 G_1 being closed; but when the water is high in the river and the fall not so great, the lower level being raised, both rings of the turbine are opened, so that at one time a large amount of water can be used with a small fall, and at another a larger fall and smaller amount of water. The centre of the upper directing portion E of each turbine is closed by a fixed bell, in which is provided a bearing for the turbine shaft, lined with strips of wood. Each turbine shaft is carried by a compound collar bearing H at the upper end, from which the whole weight of the turbine is suspended; and this bearing is carried by a pair of cast-iron girders, which are fixed across the turbine house, resting on the side walls. Another bearing between H and the rings P Q is also given, supported by a girder; it is lined with wood strips, and above it a collar is fixed to the turbine axle with clamping screws, and is adjusted so as

to be just clear of the bearing in ordinary working; but when the water pressure becomes very great a slight settlement of the turbine shaft takes place, and this collar then takes a bearing, and supports part of the weight so as to prevent there being too great friction at H. The two rings P Q, as before mentioned, regulate the supply of water. The outer ring P is suspended by six spindles, which are screwed at the top through nuts in the spur wheels R. These are geared together by a large centre wheel so that they can be all turned together by gearing from a hand-wheel (not shown in the figure), so that P can be raised or lowered to adjust the opening for the passage of the water to the outer ring. Q is raised by screwing up the outer ring until it catches projecting brackets fixed to Q, which then rises with P. As usual, the power is taken from the shaft by means of a bevel wheel above H. It will be noticed that a suction tube T is used, reaching about 4½ ft. below the bottom of the wheel, so as to allow for variations in the lower level of the stream. If this were not done, head would be lost when the water is low in the river; we must, however, leave the explanation of the action of the suction tube to a later page, merely mentioning here that as long as the lower level of the water is above the bottom of the tube the turbine may be placed at some height above the tail race, without any alteration of the effective head. Thus the lengths of the shaft and of the six spindles may be less than they would otherwise be, the reduction of weight of the former lessening the loss by friction at H.

We stated above that the velocity of whirl at discharge should be zero—that is, that the water should flow out axially; or, in this case, vertically. Now, u (fig. 32) is the total velocity of outflow, but $u \sin \beta$ is the component of the velocity that carries the water out of the wheel. However great w_2 the other component may be, no more water will flow through a wheel of a given size so long as $u \sin \beta$ is fixed. Thus, for a given wheel and given quantity of water, $u \sin \beta$ must have a fixed value; but u must be as small as possible, because one of the losses of head is $\frac{u^2}{2g}$, the energy in each pound of water as it leaves the wheel; but u is never less than $u \sin \beta$, so that the least value of u will be when $\beta = 90$ deg., and therefore $w_2 = o$.

From (7) the work done per pound

$$= \frac{c}{g}(w_1 - w_2)$$

$$= \frac{c}{g} w_1 \text{ when } w_2 = o;$$

∴ if Q be the number of cubic feet of water per second, then, neglecting friction, the horse power

$$= \frac{Q\,c\,w_1 \times 62{\cdot}5}{g \times 550} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the maximum hydraulic efficiency—

$$\eta = \frac{c\,w_1}{g\,H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

a quantity which varies between ·65 and ·9 when the turbine is running at the right speed, and none of the guide passages are closed, either wholly or partially, as this reduces the efficiency.

The ordinary parallel flow turbine is not divided into two parts, as in fig. 35, and c is then the velocity at the mean radius of the wheel vanes. In fig. 35, let c, C be the velocities at the mean radii of the blades G, G_1, and w_1 W_1 the corresponding velocities of whirl, and q_1 Q_1 the quantities of water flowing per second through E E_1, so that

$$q_1 + Q_1 = Q \text{ cubic feet.}$$

Then the horse power

$$= \frac{62{\cdot}5\,(q_1\,c\,w_1 + Q_1\,C\,W_1)}{550\,g}$$

and the hydraulic efficiency

$$= \frac{q_1\,c\,w_1 + Q_1\,C\,W_1}{g\,Q\,H}.$$

In the four last equations we have neglected the friction of the bearings, but not the frictional losses in the guide and wheel passages; the actual horse power transmitted to the bevel wheel will be therefore less than that given above, and the real efficiency will be reduced for the same reason.

Before going further, we must explain why we have not separated the theory of the turbine from the descriptions of various types of turbines. To a reader new to the subject, it will be clearer what we are aiming at if, after giving a portion of the theory, we show immediately how it may be applied, and the change from theory to description will, we hope, make these articles far more readable. We should not have made this apology had we not known that we were not following the usual practice of writers on this and similar subjects.

CHAPTER IX.

Classification of Turbines.

All turbines belong to one of two classes, called reaction and impulse turbines. In the former, when working at full power, all the guide and wheel passages are filled with water, and the turbine is said to be drowned, and the velocity of flow in one part can be determined when that in any other part is known; or, to put it mathematically, if A_1 A_2 A_3 be the cross-sections of the stream at any point, and v_1, v_2, v_3 the velocities perpendicular to those sections, then $v_1 A_1 = v_2 A_2 = v_3 A_3$.

Wherever the quantity of water is large, and the fall moderate, a reaction turbine is generally used.

An impulse turbine is not drowned, the buckets are not filled, and in some cases, in which there is only partial admission, each bucket is empty during part of a revolution.

Fig 36.

Air is required in the buckets, so that the pressure may always be that of the atmosphere, ventilating apertures being made in their sides for this purpose. A suction tube cannot be used with this class of turbine, which is suitable for high falls and moderate or small quantities of water.

Thus a reaction turbine may be made for a fall of 14 ft., and 200 cubic feet of water per second; but an impulse turbine is required for a fall of 600 ft., and 20 cubic feet of water per second. In the former case an impulse turbine might be used in place of the reaction turbine, but the high

number of revolutions required for the reaction turbine in the latter case, where the fall is high, makes it necessary to use an impulse turbine; for with a small quantity of water, a turbine that is filled must be of small diameter, and as the velocity of rotation at the radius at which inflow takes place is not less than $\cdot 45 \sqrt{2 g H}$ for a reaction turbine, and is generally greater, the number of revolutions would become inconveniently high when H is large.

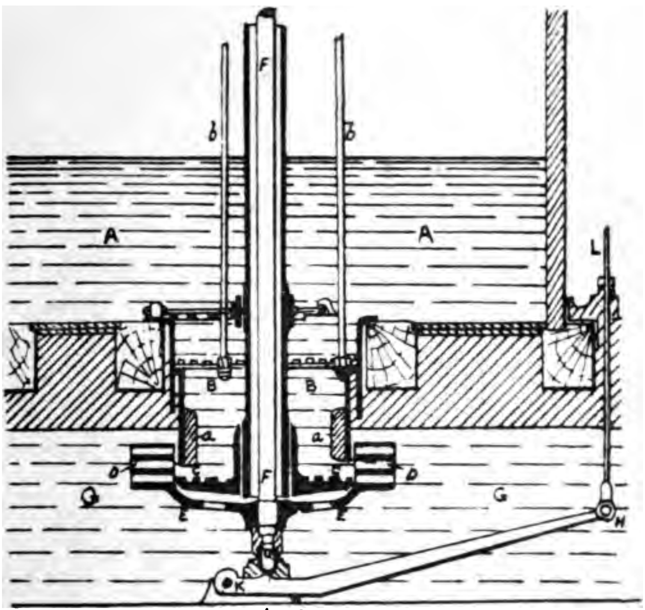


Fig 37.

Reaction and impulse turbines may be again divided into three classes—radial, axial, combined or mixed flow. In the first the water flows outwards or inwards; the second we have already described; and in the third the water enters approximately radially, and leaves axially. The axes may be in all cases vertical or horizontal. Fig. 36 shows a section perpendicular to the axis, and through the guide apparatus

and wheel of a radial outward-flow or Fourneyron turbine. At B are the guide vanes and at D the wheel vanes, the arrow showing the direction of rotation. Fig. 37 is a vertical section of this type of wheel. A is the tank or penstock, B the supply cylinder, which consists of two concentric tubes; the upper is fixed, the lower slides within it like the inner tube of a telescope, and is raised or lowered by the rods *b*; near the upper edge of the inner tube is a leather collar to make the joint between it and the outer tube water-tight. The lower part *a* of the inner tube acts as a regulating sluice for all the orifices at once. It has fixed to its internal surface wooden blocks, so shaped as to round off the turns in the course of the water towards the orifices.

The bottom of the supply cylinder is formed by a fixed disc C, which is supported by hanging at the lower end of a fixed vertical tube enclosing the shaft. This disc carries the guide blades.

D are the vanes of the wheel, which are in this case divided into three sets or horizontal layers, by two intermediate crowns or horizontal ring-shaped partitions. The

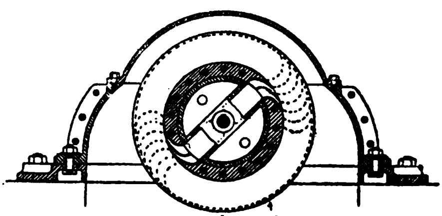


FIG. 38.

object of this is to secure that the passages shall be filled by the stream at three different elevations of the sluice, and so to lessen the loss of efficiency which occurs when the opening of the sluice is small. E is the disc of the wheel, F its shaft, G the tail race. K H is a lever which supports the step of the pivot, and is itself supported by fixed bearings at K and by the rod L, which can be raised or lowered by a screw, so as to adjust the wheel to the proper level. The turbine shown is a reaction turbine, but its mechanical construction is the same if it works as an impulse turbine. In the latter case, however, the turbine is not drowned, the discharge taking place above the level of the tail race. A

suction tube is never used with any form of outward-flow turbine.

Fig. 38 shows an impulse outward-flow turbine. This type is manufactured by Messrs. Reiter and Co., of Winterthur,

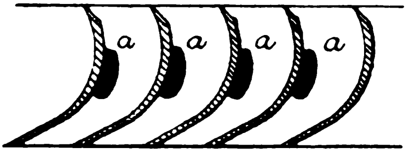


Fig. 39.

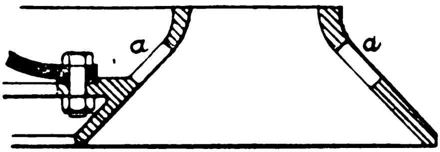


Fig. 40.

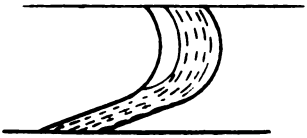


Fig 41.

for falls up to 650 ft. The wheel is of cast iron, and the inflow takes place at the opposite ends of a diameter, the two guide passages at either end being made of wrought iron. The valve which regulates the opening of these is made of gun metal, and is keyed to a spindle which can be rotated through the small angle required to close the passages, by

hand or by a governor. The following are particulars of two turbines similar to the above :—

	I.	II.
Inner diameter of wheel......	11·81 in.	11·81 in.
Outer diameter of wheel......	16·14 in.	14·84 in.
Number of vanes	45 cast iron	84 wrought iron
Number of revolutions per minute	583	928
Quantity of water per minute	4 52 c. ft.	5·085 c. ft.
Head of water.....................	129 ft. 1·6 in.	131 ft. 5·19 in.
Available power per minute.	36,440 ft.-lb.	53,806 ft.-lb.
Power measured on brake....	16,498 ft.-lb.	28,809 ft.-lb.
Efficiency per cent	45·27	53·5

We have already described an axial-flow reaction turbine, and an impulse turbine of the same class so closely resembles it that we do not propose to give a general view of one of these until a later chapter.

Figs. 39 and 40 show sections through the wheel, *a a* being the ventilating apertures. The great breadth of the wheel in fig. 40 will be at once noticed. It is so made to prevent the passages becoming filled at outflow (fig. 41), for if this took place the wheel would not work as an impulse turbine should do; for while in reaction turbines continuity of flow is a necessity, in impulse turbines it must not take place, for the pressure must always be that of the atmosphere, for which reason the ventilating apertures are provided.

Fig. 42 is an outline drawing of an inward-flow turbine, with suction tube and cylindrical sluice at the bottom; the lower part of the figure is a sectional plan through guide apparatus and wheel. These two latter are marked A and B. C is the suction tube, while D is the regulating sluice, by no means an economical method of regulation, because as the passage at outflow from the sluice is diminished the water flows more slowly through the whole wheel; it has therefore a smaller velocity of whirl at entry, and its entry is attended with shock; again, on leaving the wheel it still has some tangential velocity left, because the velocity of the water relative to the wheel is less than when the sluice is fully raised. The velocity of the stream passing away from D is also increased, so that we have three important causes of loss. How these affect the efficiency will be better understood when the theory of radial turbines has been studied in a subsequent chapter. E are the rods by means of which the sluice can be lifted. The shaft is carried by a collar

bearing above. Arrows show the direction of flow of the water; the letters F F and the dotted curves below them represent the vanes of an inward and parallel flow wheel, and do not refer to the inward-flow turbine. The water is turned from a radial to an axial direction, and hence the name "mixed-flow" for this type. The inward-flow turbine

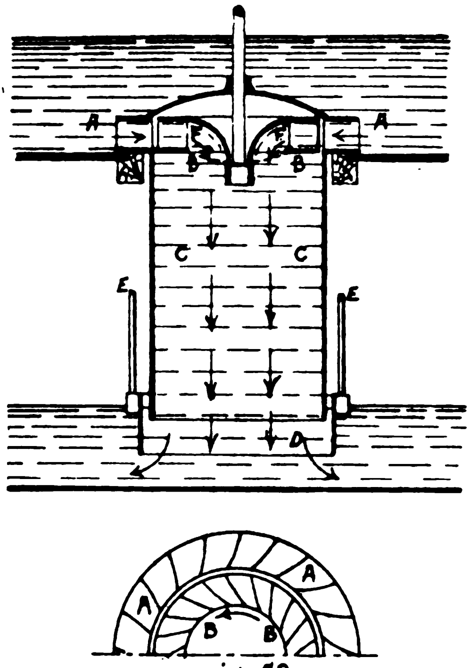


Fig. 42.

of the late Professor James Thompson, who was the inventor of this type, is shown in figs. 42*a*, 42*b*, and 42*c*. The first shows the wheel, the part B being in section; its sides W are conical (fig. 42*b*). The water enters at O (fig. 42*c*), and flows round in both directions, and enters in the manner shown by the arrows. It seems probable that had the casing been made like that of the spiral chamber or volute

of a centrifugal pump it would have been better, because the direction of motion here turns through almost 180 deg. at the top of the wheel casing. The spiral casing is shown by Weisbach in his Mechanics. Instead of the area being greatest at O and least at the opposite end of the diameter, it decreases from O uniformly round the circumference, the water flowing in the direction of the hands of a clock, so that no alterations of direction take place. The guide vanes D are hinged at E, and can be moved by links C, so that the passage to the wheel is increased or contracted according as the guide vanes move in the direction of the hands of a

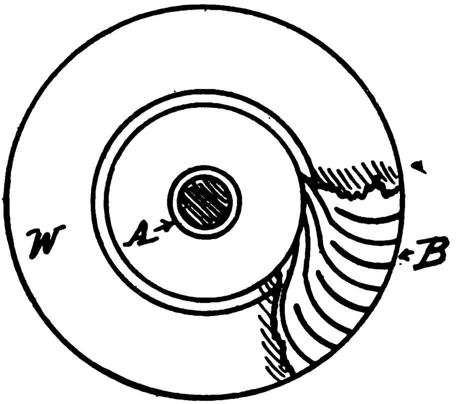


FIG. 42*a*

watch, or the opposite. This wheel is very efficient both when working at full and reduced powers. Alteration of the guide vane angle has been very successful both for inward and axial turbines, although some energy must be lost by shock at entry at part gate, and as the quantity of water flowing through the wheel is then less, the relative velocity of discharge is less, and as this has a backward direction which, with the velocity of the inner circumference, would give a radial direction of outflow at full gate, the water has a forward velocity of whirl at part gate, which reduces the efficiency, as will be shown in the "Theory of Radial Turbines." The water finally leaves the wheel in directions parallel to the axis, and passes out of the casing down the draught tubes P. The term "part gate" means

that the quantity of water flowing to the wheel is reduced; the expression is obtained from America, and probably came from the old-fashioned sluice gate of a common water-wheel. The great advantages of an inward over an outward flow turbine are, firstly, plenty of space outside the wheel for the guide apparatus; secondly, a wheel of less weight for a

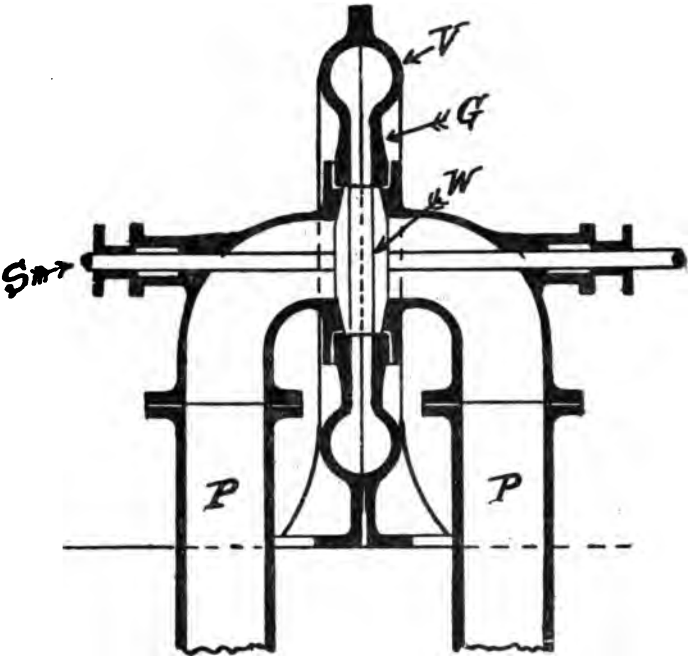


FIG. 42*b*.

given velocity of circumference at inflow (see "Theory of Radial Turbines"); thirdly, a suction tube may be used.

Fig. 43 shows the "Victor," of American design. The inflow takes place at A and the outflow at B.* This turbine is at work on the river Greta, that flows through Keswick, and supplies the power for the electric light

* Fuller particulars of this turbine will be given further on.

station. The wheel is 20 in. in diameter, the head is 20 ft., and the number of revolutions per minute 273. The driving shaft is horizontal, and as it is 16 ft. above the tail-race, a suction tube of wrought iron, 3 ft. diameter, is used. The regulation is effected by opening and closing a cylindrical

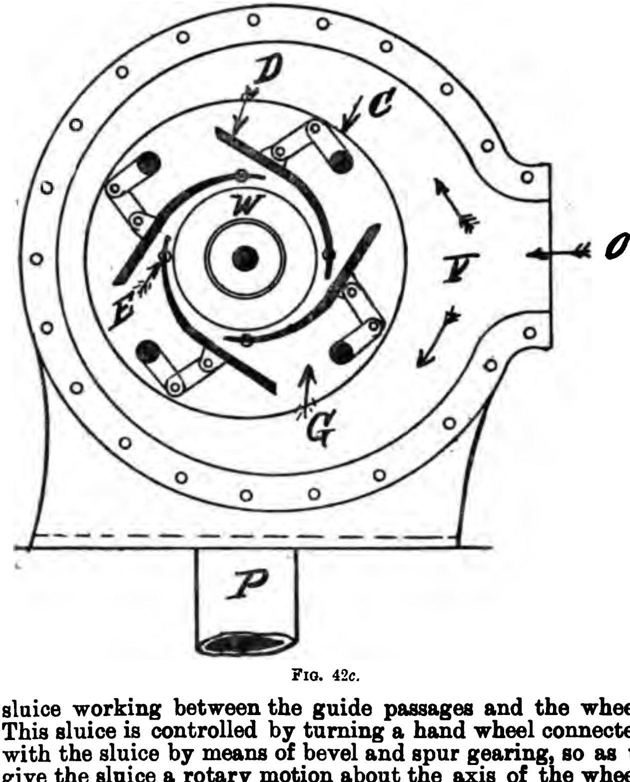
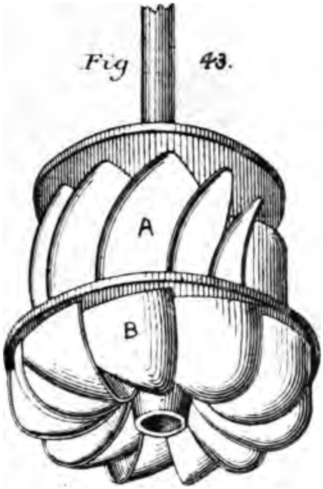


FIG. 42c.

sluice working between the guide passages and the wheel. This sluice is controlled by turning a hand wheel connected with the sluice by means of bevel and spur gearing, so as to give the sluice a rotary motion about the axis of the wheel. Fig. 44 gives an external side elevation of the whole arrangement. To the right is the turbine casing, showing the outside of the guide apparatus, and above it are the bevel wheels and shafting for the regulation. The above description is from the Proceedings of the Institution of

Civil Engineers, vol. cii. No detailed description is given of the sluice, nor any section of the turbine, but this method

Fig 43.



of regulation is shown in fig. 45, although this may not be the exact arrangement for the above turbine. A is the wheel, B the guide vanes, and between them is the cylindrical

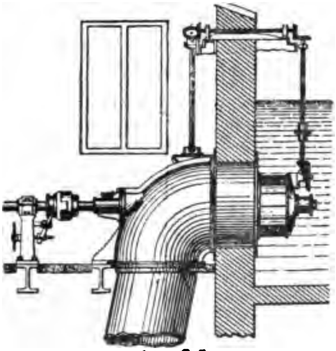


Fig 44.

sluice, which has spur-wheel teeth at the right-hand end (fig. 44), which gearing, with a pinion on the short shaft that carries the last bevel wheel of the regulation shafting, enables the sluice to be turned by the hand wheel. The pinion and sluice are hidden by the casing in fig. 44.

We have frequently stated that a reaction turbine must be filled with water to act correctly, and it is best to have a number of sluices, so that no guide passages are partially closed, but as less and less power is required more passages are closed, each by its own sluice. The above arrangement, however, partially closes all the guide passages at once, and the efficiency at lower powers is consequently reduced, as the following will show :—

Cubic ft. of water per second.	Efficiency of turbine and dynamo per cent.
23·48	75·00
14·83	58·65
11·29	43·80
9·68	37·90

Another method of regulation of the inward-flow turbine is by a cylindrical sluice between the guide passages and the wheel, the motion of which is parallel to the axis of

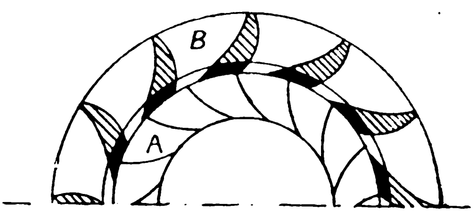


Fig. 45.

the wheel, just as in the Fourneyron outward-flow turbine, fig. 37, and when the wheel is divided by partitions transverse to the axis, as in the case of the Fourneyron turbine, the loss of efficiency will not be so great. As an example, a Hercules turbine of the mixed-flow type had an efficiency, as measured by the brake, of not less than 70·9 per cent when the gate opening was ·379 of the whole, as against 85·8 at full gate, while a "Humphrey" turbine regulated in a similar manner to the "Victor" turbine had an efficiency of 61 per cent when the sluice was about half open, against nearly 82 per cent when it was fully open.

CHAPTER X.

THE SUCTION TUBE.

WE have mentioned that the suction tube enables a reaction turbine to be placed in a higher and consequently more convenient position than it otherwise would be, shortening the shaft and regulating rods, and sometimes, as in the case of the Victor turbine, allowing bevel wheels to be dispensed with, and pivot friction exchanged for journal friction, which latter has been shown by the latest experiments to be less than the former. We now intend to explain the action of the tube.

Fig. 46 shows a tube A B, in which is a piston P, C D being the upper level and E F the lower level of the water. Now, suppose the atmospheric pressure is replaced by the pressure of two columns of water A C and K B; then the pressure on P is evidently unaltered. But if we express the pressure on P in feet of water, the pressure on its upper surface is A P, while that on its under surface is P P; the effective pressure is therefore A K, and

$$\begin{aligned} AK &= AC + CK \\ &= KB + CK \\ &= BC \end{aligned}$$

so that as long as P is below K L where K B is the height of the water barometer, the head is unaltered.

Now, with a given turbine with a given resistance to overcome, and with a given head, no matter how that head is obtained, whether by water pressure or atmospheric pressure, a certain number of revolutions will be obtained, and a certain quantity of water will be used. This is evident, because the state of affairs at the turbine is unchanged, for the pressure at every point is unchanged, as shown above; and if the forces and the sphere in which they act are unchanged, the motions will also be unchanged.

This, then, explains how a turbine with a draught or suction tube can be placed above the tail race. The height is theoretically limited to 34 ft., but practically to much less. More accurately, if v_3 be the velocity of flow from the suction tube, theory would give

$$h_3 = 34 - \frac{v_3^2}{2g}$$

where h_3 is the above height.

In practice, however, in the above equation the following quantities should be substituted instead of 34 for the corresponding diameters of suction tube. This table is given by Meissner.

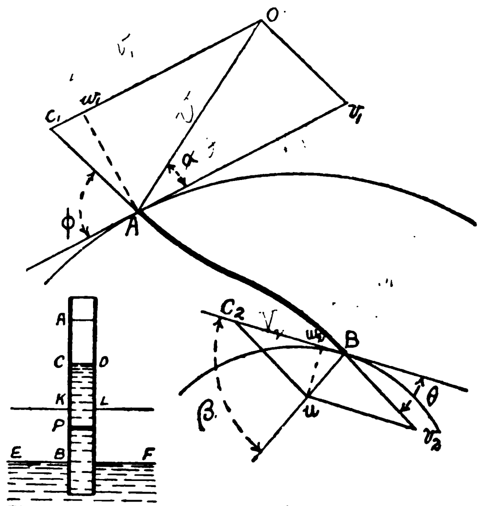


Fig. 46. Fig 47.

Diameter of tube in feet.		$h_3 + \frac{v_3^2}{2g}$
·49	....	31·16
·98	... .	29·52
1·6	....	27·88
2·3	....	27·88
3·3	....	26·24
4·9	....	19·68
6·5	....	14·76
8·2	....	13·77
9·8	....	12·46
11·5	....	11·15
13	....	9·84

CHAPTER XI.

THEORY OF RADIAL-FLOW REACTION TURBINES.

IN figs. 47, 48, the former of which refers to an inward-flow and the latter to an outward-flow wheel, let A B be a vane and O c_1 A v_1, B c_2, u v_2 the parallelograms of velocities of the water at entry into and exit from the wheel; O A is the absolute velocity at entry, which is supposed to take place without sudden change of direction or velocity, so that O v_1 is the relative velocity of inflow, while O c_1 is the velocity of the wheel A. Drop A w_1, a perpendicular on O c_1, then O w_1 is the tangential velocity of the water at entry, or the velocity of whirl. The angles α and ϕ should also be noticed. Let O A, O c_1, O v_1, O w_1 be v, c_1, v_1, and w_1.

Then
$$\left.\begin{aligned} w_1 &= v \cos \alpha \\ c_1 - w_1 &= v_1 \cos \phi \\ c_1 \sin \phi &= v \sin (\alpha + \phi) \\ v_1 \sin \phi &= v \sin \alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Similarly at the point B, B u, B c_2, B v_2, B w_2 are the absolute velocity u, the wheel velocity c_2 at B, the relative velocity v_2 of the water at outflow, and its velocity of whirl w_2. Notice also the angles β and θ.

Then
$$\left.\begin{aligned} w_2 &= u \cos \beta \\ c_2 - w_2 &= v_2 \cos \theta \\ c_2 \sin \theta &= u \sin (\theta + \beta) \\ v_2 \sin \theta &= u \sin \beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The work done by a weight of water W having the above velocities is

$$\frac{W}{g} (w_1 c_1 - w_2 c_2);$$

and if β is a right angle, $w_2 = O$; $\therefore$ the work done is $\frac{W}{g} w_1 c_1$, and fig. 49 shows that $c_2 = v_2 \cos \theta$.

The reader will remember that in the case of the axial flow we showed that β should be a right angle if maximum efficiency was to be obtained, and the same reasoning will apply here; one of the losses of energy per pound is $\frac{u_2}{2g}$ caused by the water leaving the wheel with a velocity u. This, then, should be as small as possible. Now, for a wheel of a given size the quantity of water flowing out of the

wheel depends on $(u\,w_2)$, the radial component of $(B\,u)$; hence for a given quantity of water per second the waste of energy will be least when $B\,u = u\,w_2$ or when β is a right angle, which is, therefore, one condition for maximum efficiency.

The hydraulic efficiency is, therefore,

$$\eta = \frac{c_1\,w_1}{g\,H} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

because $\frac{c_1\,w_1}{g}$ is the actual work done per pound, and H is the work it would do if there were no waste of energy.

This efficiency is called hydraulic because it takes into account hydraulic losses, but not friction of shafting. These losses are caused by the friction of a supply pipe if there is one, from friction and curvature of guide and wheel

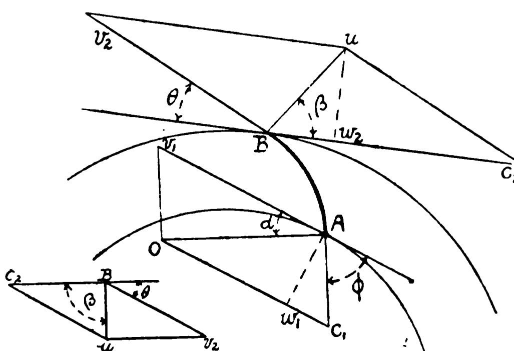


Fig. 49. Fig. 48.

passages, from leakage, and shock at entry into the wheel, and, if a suction tube is used, from shock on leaving the wheel and friction in the tube. The lower edge of the tube, or sluice if there is one at its end, is another cause of waste, which may be reduced by rounding the edge, and thus lessening the sudden change of direction and velocity at this point. In designing we must try to arrange that shock shall not occur at entry and exit from the wheel, although it will probably occur to a greater or less extent in practice; so at least experiments show. Whenever in its flow the stream encounters the edge of a vane a retardation takes place, which may be lessened by making these edges thinner,

but which cannot be prevented. Like all frictional losses, these are exceedingly variable, but it is convenient for purposes of design to give them values depending on certain coefficients and velocities, the coefficients being given average values, deduced from experiment.

All the losses of head caused by the guide passages may be represented by the quantity $F \frac{v^2}{2g}$ so that F is a coefficient of resistance referred to the velocity of discharge from the guide passages. There is great difference of opinion as to the value of F, which appears to vary from ·05 to above ·2. In the following we shall take it as ·125.

The loss by leakage and by friction of wheel and curvature of path through the wheel is represented by $F_2 \frac{v_2^2}{2g}$ because v_2 is the relative velocity of discharge from the wheel passages. Hänel gives F_2 from ·1 to ·2, and in the following we shall take ·2 as its value. If there is no suction tube, the remaining loss is $\frac{u^2}{2g}$ from unutilised energy, and if there is one, the loss of head is $\frac{v_3^2}{2g}(1 + F_3)$, the coefficient F_3 depending upon the lower edge of the tube. This varies more than any of the other coefficients, but we shall assume the minimum value obtained by experiment, supposing that the tube is well rounded at the bottom.

If L represents the loss of head,

$$L = \cdot 125 \frac{v^2}{2g} + \cdot 2 \frac{v_2^2}{2g} + 2 \frac{v_3^2}{2g} \text{ with a suction tube} \quad . \quad (15)$$

and $$L = \cdot 125 \frac{v^2}{2g} + \cdot 2 \frac{v_2^2}{2g} + \frac{u^2}{2g} \text{ without one} \quad . \quad . \quad (16)$$

Let Q = cubic feet per second of water

$$= a\,v = a_2\,v_2 = a_3\,v_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

so that a, a_2, a_3 are the cross-sections of the stream at discharge from guide passages, wheel passages, and suction tube.

The hydraulic efficiency $\eta = \frac{2 c_1 w_1}{2 c_1 w_1 + 2 g L}$ . . . (18)

because $\frac{c_1 w_1}{g}$ is the useful work per lb., and $\frac{c_1 w_1}{g} + L = H$ (19) for the potential energy of each pound is expended in useful work and losses.

$$\therefore \eta = \frac{2\frac{r_1}{r_2} c_2 v \cos \alpha}{2\frac{r_1}{r_2} c_2 v \cos \alpha + \cdot 125\, v^2 + \cdot 2\, v_2{}^2 + 2 v_3{}^2}$$

with suction tube; also

$$2 g \mathrm{H} = 2 \frac{r_1}{r_2} c_2 v \cos \alpha \times \cdot 125\, v^2 + \cdot 2 v_2{}^2 + 2 v_3{}^2$$

$$= v_2{}^2 \left\{ 2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha + \cdot 125 \left(\frac{a_2}{a}\right)^2 + \cdot 2 + 2\left(\frac{a_2}{a_3}\right)^2 \right\}$$

$$v_2 = \sqrt{\frac{2 g \mathrm{H}}{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha + \cdot 125 \left(\frac{a_2}{a}\right)^2 + \cdot 2 + 2\left(\frac{a_2}{a_3}\right)^2}}\,; \quad (20)$$

$$\eta = \frac{2 v_2{}^2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha}{2 g \mathrm{H}}$$

$$= \frac{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha}{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha + \cdot 125 \left(\frac{a_2}{a}\right)^2 + \cdot 2 + 2\left(\frac{a_2}{a_3}\right)^2}\,; \quad . \quad (21)$$

If no suction tube is used, remembering that $u = v_2 \sin \theta$, and taking L from (16),

$$v_2 = \sqrt{\frac{2 g \mathrm{H}}{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha + \cdot 125 \left(\frac{a_2}{a}\right)^2 + \cdot 2 + \sin^2 \theta}}\,; \quad (22)$$

$$\eta = \frac{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha}{2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha + \cdot 125 \left(\frac{a_2}{a}\right)^2 + \cdot 2 + \sin^2 \theta}\,; \quad . \quad . \quad (23)$$

As an example of the application of the above equations, we shall now give the necessary calculations for the design of a radial inward-flow turbine to utilise a fall of 13 ft., and 113 cubic feet of water per second. We are able to assume certain quantities, and we shall take—

$$\frac{a_2}{a} = 1 \cdot 15\,; \qquad \theta = 15^\circ\,; \qquad \alpha = 12^\circ\,;$$

$$\frac{r_1}{r_2} = 1 \cdot 169\,; \quad \frac{a_2}{a_3} = 0 \cdot 25.$$

$$2 \frac{r_1}{r_2} \frac{a_2}{a} \cos \theta \cos \alpha = 2 \times 1 \cdot 169 \times 1 \cdot 15 \times \cdot 966 \times \cdot 978 = 2 \cdot 54.$$

$$\eta \text{ from } (21) = \frac{2{\cdot}54}{2{\cdot}54 + {\cdot}165 + {\cdot}2 + {\cdot}125} = {\cdot}84 \text{ nearly.}$$

$$v_2 \text{ from } (20) = \sqrt{\frac{2\,g\,\mathrm{H}}{3{\cdot}03}} = {\cdot}57\ \sqrt{2\,g\,\mathrm{H}.}$$

$$= 16{\cdot}4 \text{ ft. per second.}$$

The velocity of wheel at inner radius $= c_2$, and from (13)

$$c_2 = v_2 \cos\theta = 16{\cdot}4 \times {\cdot}966 = 15{\cdot}8$$

$$\mathrm{Q} = a_2\,v_2\ ;\ a_2 = \frac{113}{16{\cdot}4} = 6{\cdot}89 \text{ square feet}$$

$$a = \frac{a_2}{1{\cdot}15} = 5{\cdot}99 \text{ square feet}$$

$$v = 1{\cdot}15\,v_2 = 1{\cdot}15 \times 16{\cdot}4 = 18{\cdot}85$$

$$c_1 = \frac{r_1}{r_2}\,c_2 = 1{\cdot}169 \times 15{\cdot}8 = 18{\cdot}6.$$

The parallelogram $\mathrm{O}c_1\,a\,v_1$ in fig. 47 will give ϕ by graphic construction, but it may be calculated thus—

$$c_1 \sin\phi = v \sin(\alpha + \phi)$$

$$\tan\phi\,(c_1 - v\cos\alpha) = v\sin\alpha$$

$$\tan\phi = \frac{v\sin\alpha}{c_1 - v\cos\alpha}\ ;\ \text{and } \alpha = 12^\circ$$

$$= \frac{18{\cdot}85 \times {\cdot}208}{18{\cdot}6 - 18{\cdot}85 \times {\cdot}978}$$

$$= 19{\cdot}6\ ;\ \phi = 87^\circ\ 5'.$$

Let b, b_2 be the breadths of the guide and wheel passages, measured parallel to the axis.

$$\mathrm{K}\,b\left\{2\,\pi\,r_1\sin\alpha - n\,t - n_1\,t_1\frac{\sin\alpha}{\sin\phi}\right\} = a \quad . \quad . \quad (24)$$

$$\mathrm{K}\,b_2\ \{\,2\,\pi\,r_2\sin\theta - n_1\,t_2\,\} = a_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where n, n_1 are the numbers of vanes in guide apparatus and wheel; t, t_1, t_2 are the thicknesses of the vanes at the outlet of the guide passages at radii $r_1\ r_2$ of the wheel. We think the reader will at once see that $n\,t$ in (24) represents the obstruction caused by the guide vanes, and $n_1\,t_2$ in (25) that due to the wheel vanes, while $n_1\,t_1\,\frac{\sin\alpha}{\sin\phi}$ in (24) is that caused by the wheel vanes at entry. If n and n_1 were equal at n points of the revolution, the guide vanes would be opposite the wheel vanes, and at those instants the

quantity $n_1\ t_1\ \frac{\sin\alpha}{\sin\phi}$ might be omitted. To obviate this variation in the value of a—

$$n_1 = n + 1, \text{ or } n + 2.$$

K is a coefficient of contraction, about ·9 for radial turbines; r_1, r_2 cannot be fixed by any mathematical formula.

We shall here take $r_2 = 4$ ft., whence

$$r_1 = 4 \times 1{\cdot}169 = 4{\cdot}676 \text{ ft.};$$
$$t = \tfrac{3}{16} \text{ in.} = {\cdot}187 \text{ in.};$$
$$t_1 = t_2 = {\cdot}25 \text{ in.}$$

These thicknesses are tapered off at the ends, but it is safer to take the full thickness when the vanes are of wrought iron. As a general rule t and t_1 are from $\frac{1}{2}$ in. to $\frac{5}{8}$ in. for cast iron, and $\frac{1}{4}$ in. to $\frac{3}{8}$ in. for wrought iron at the ends, though in the latter case they are sometimes less than $\frac{1}{8}$ in. thick.

$$n = 40;\ n_1 = 41.$$

Applying (24)—

$${\cdot}9\,b = \frac{5{\cdot}99}{2\pi \times 4{\cdot}676 \times {\cdot}208 - \frac{40 \times {\cdot}187}{12} - \frac{41 \times {\cdot}25 \times {\cdot}208}{12 \times {\cdot}999}}$$

$$b = 1{\cdot}255 \text{ ft.}$$

and from (25)

$$b_2 = \frac{6{\cdot}89}{\left(2\pi \times 4 \times {\cdot}258 - \frac{41 \times {\cdot}25}{12}\right) \times {\cdot}9} = 1{\cdot}355 \text{ ft.}$$

These, with the exception of a_3, are all the necessary theoretical calculations—

$$a_3 = \frac{a_2}{{\cdot}25} = 6{\cdot}89 \times 4 = 27{\cdot}56,$$

corresponding to a diameter of 6 ft. nearly.

In practice, however, the diameter of tube is often about equal to the external diameter of the wheel, although theory would require that the velocity of the water on leaving the wheel should be unchanged until it has taken an axial direction, and that the suction tube should gradually increase in diameter, so that shock may be avoided. If we suppose that the diameter is equal to $2\,r_1$, then

$$a_3 = 6{\cdot}86, \text{ and } \frac{a_2}{a_3} = {\cdot}1, \text{ very nearly.}$$

The head lost by shock will be

$$\frac{1}{2g}(u - v_3)^2 = \frac{v_2^2}{2g}\left(\sin\theta - \frac{a_2}{a_3}\right)^2$$

$$= \frac{v_2^2}{2g}(\cdot 1588)^2 \quad = \frac{\cdot 025}{2g} v_2^2.$$

In addition to this there will be a loss

$$= \frac{2 v_3^2}{2g},$$

caused by the lower edge of the suction tube and the unutilised energy; but

$$\frac{2 v_3^2}{2g} = \frac{\cdot 02 \, v_2^2}{2g},$$

$\therefore$ total loss after discharge from wheel is

$$\frac{\cdot 045 \, v_2^2}{2g},$$

Hence in the equation for the hydraulic efficiency, instead of ·125, the last term in the denominator, we should have ·045. Then—

$$\eta = \frac{2 \cdot 54}{2 \cdot 54 + \cdot 165 + \cdot 2 + \cdot 045} = \frac{2 \cdot 54}{2 \cdot 95} = \cdot 86.$$

This would slightly alter all the above calculations, but for all practical purposes we may allow them to remain as they are, for it must be remembered that the coefficients of resistance—F, F_2, F_3—are quantities whose values cannot be absolutely fixed.

To summarise the above method of calculating a numerical example, the quantities given are Q and H, and we assume

$$\frac{a_2}{a}, \quad \alpha, \quad \theta, \quad \frac{r_1}{r_2}, \quad \frac{a_2}{a_3}.$$

η is then obtained from (21), v_2 from (20), c_2 from (13), then a_2 and a from (17), which also gives v, and

$$c_1 = c_2 \frac{r_1}{r_2}.$$

ϕ can be found graphically by fig. 47, or from

$$\tan \phi = \frac{v \sin \alpha}{c_1 - v \cos \alpha}.$$

r_2 or r_1 may now be assumed, and the other calculated from the known ratio $\frac{r_1}{r_2}$. t, t_1, t_2 must now be assumed from the above rules, and n and n_1, experience being the sole guide; and b and b_2 must be found by (24) and (25).

We may greatly simplify the above by assuming a probable value for the hydraulic efficiency. The mechanical efficiency, as obtained by a brake on the turbine shaft, should not be less than 80 per cent for a wheel of average size, and this differs from the hydraulic efficiency by about 3 per cent, due to shaft friction. Suppose, then, we have $\eta = \cdot83$; then

$$\frac{c_1 w_1}{g} = \cdot83\,\text{H}$$

$$= \frac{1}{g}\frac{a_2}{a}\frac{r_1}{r_2} c_2 v_2 \cos \alpha = \frac{1}{g}\frac{a_2}{a}\frac{r_1}{r_2} v_2{}^2 \cos \alpha \cos \theta$$

$$\therefore v_2 = \sqrt{\frac{\cdot83 \times 2g\,\text{H}}{2\frac{a_2}{a}\frac{r_1}{r_2}\cos \alpha \cos \theta}}$$

$$= \cdot91\sqrt{\frac{2g\,\text{H}}{2\frac{a_2}{a}\frac{r_1}{r_2}\cos \alpha \cos \theta}} \text{ nearly.} \quad . \quad . \quad . \quad (26)$$

and this will enable us to proceed as before.

CHAPTER XII.

Theory of Axial-flow Reaction Turbine.

In a radial-flow turbine every particle of water enters and leaves the wheel at the same radii r_1 r_2, but in an axial turbine some enters at a less distance from the axis than others. It is, however, sufficient for all practical purposes to treat every particle as if it entered and left at the mean radius of the wheel, which we shall denote by r. Certain small modifications as to vane angles will afterwards have to be considered, but these may be left for the present. All the above formulæ may be used if we make $r_1 = r_2 = r$, $\text{K} = 1$, and $c_1 = c_2 = c$ the velocity at the mean circumference; then the work done per pound is

$$\frac{c}{g}(w_1 - w_2) = \frac{c\,w_1}{g} \text{ when } w_2 = o.$$

This gives us

$$v_2 = \cdot91\sqrt{\frac{2g\,\text{H}}{2\frac{a_2}{a}\cos \alpha \cos \theta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26\text{A})$$

$$b = \frac{a}{2\pi r \sin\alpha - nt - n_1 t_1 \dfrac{\sin\alpha}{\sin\phi}}; \quad . \quad . \quad . \quad . \quad (24\text{A})$$

$$b_2 = \frac{a_2}{2\pi r \sin\theta - n_1 t_2}; \quad . \quad . \quad . \quad . \quad (25\text{A})$$

generally $b = b_2$, but this is not necessary.

We shall now give a numerical example: An axial-flow turbine is required to utilise a fall of 12·1 ft., = H, and 212 cubic feet of water per second.

Assume $\alpha = 20°$, $\theta = 17°$, $\frac{a_2}{a} = \cdot97$. In practice, α varies between 15° and 17° for high falls and small quantities of water, and from 20° to 24° for large quantities and low falls. $\frac{a_2}{a}$ may be taken from ·75 to 2, but is generally not far from unity.

$$\cos\alpha = \cdot936,\ \cos\theta = \cdot956,$$

$$\sin\alpha = \cdot342,\ \sin\theta = \cdot292,$$

$$v_2 = 19\cdot2, \text{ from } (26\text{A})$$

$$v = \frac{a_2}{a} v_2 = 18\cdot6,$$

$$c = v_2 \cos\theta = 18\cdot35,$$

$$\tan\phi = \frac{v \sin\alpha}{c - v\cos\alpha} = 6\cdot36;\ \phi = 81°\ 4',$$

$$a = \frac{Q}{v} = \frac{212}{18\cdot6} = 11\cdot4 \text{ square feet},$$

$$a_2 = \cdot97 \times 11\cdot4 = 11\cdot05 \text{ square feet.}$$

If $r = 4\cdot925$, the revolutions per minute are 35·6.

Neglecting vane thicknesses,

$$b = \frac{a}{2\pi r \sin\alpha}$$

$$= 1\cdot08 \text{ ft.; if } r = 4\cdot925 \text{ ft.}$$

$$b_2 = \frac{a_2}{2\pi r \sin\theta} = 1\cdot225 \text{ ft.}$$

The greater the value of r, the less are b and b_2, and, as will be subsequently shown, the less are the variations of vane angles. But, on the other hand, the wheel takes up more space and becomes heavier; b must be some fraction of

r not absolutely fixed in practice, and as $b\,r$ is proportional to a, so r must be proportional to $\sqrt{a}$, although, again, in practice we meet with great variety of proportion.

The best rules are as follow: When a is less than 2 square feet, r varies from $1{\cdot}5\sqrt{a}$ to $2\sqrt{a}$, and when a is more than 2 and less than 16, it lies between $1\frac{1}{4}\sqrt{a}$ and $1\frac{1}{2}\sqrt{a}$, while when a is more than 16 it varies between $\sqrt{a}$ and $1\frac{1}{4}\sqrt{a}$.

The above approximate calculations should be made to decide on a probable value of the radius r before making the more lengthy calculations required by (24A) and (25A). Taking $t = \frac{1}{4}$ in. $= \frac{1}{48}$ ft. $= t_1 = t_2$, $n = 40$, $n_1 = 41$, $\sin\phi = {\cdot}9878$, and applying (24A) and (25A),

$$b = 1{\cdot}15 \text{ very nearly}$$
$$b_2 = 1{\cdot}35.$$

The depth of guides and wheel measured parallel to the axis should be $1\frac{1}{4}$ ft. to $1{\cdot}1$ ft., although there is no mathematical rule for this. The deeper the wheel the greater the weight, but the more gradual the change of direction of the flow in the wheel.

If there is known to be more than a sufficient supply of water, the problem may take a slightly different form; the horse power and fall may be given. Then we may assume a low efficiency, say ·75, and from this calculate the quantity of water per second that is needed; after this

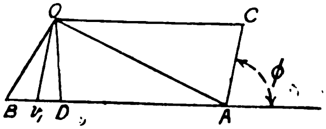


FIG. 50.

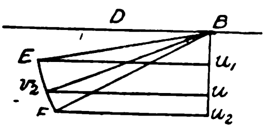


FIG. 51.

the problem is the same as those above. The turbine will probably be a little more powerful than is necessary.

Having settled the values of $\alpha\ \phi\ \theta$ for the mean radius, it is necessary to find the corresponding quantities for the outer and inner radii. We shall first suppose α constant and equal to O A B, fig. 50. Then O A is v, O c is c, the wheel velocity at the mean radius, which also equals A v_1; but at the outer radius A B is the wheel velocity, and at the inner radius A D, so that O B A, O D A are the theoretical values of ϕ. In fig. 51, B E $=$ B v_2 $=$ B F $= v_2$, while u_1 E, $u\,v_2$, u_2 F are the wheel velocities at outer, mean, and inner radii. Hence E B D,

F B D are the values of θ at the outer and inner radii. In practice, however, α is not constant, but decreases from the inner to the outer radius, because for simplicity of construction it is made so that a radial line perpendicular to the axis will touch the vane from the inner to the outer radius, so that the vane forms a helical surface of variable pitch.

When a wheel is cast the spaces between the vanes are cored, each with a separate core, so that there ought to be very little difficulty in giving the vanes the correct angles, if this is theoretically possible. In the wheel designed above it will be well to see how far theory and practice differ from one another with regard to these angles. Let a subscript 1 refer to dimensions at the outer radius, and

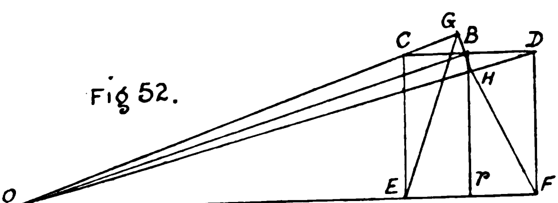


Fig 52.

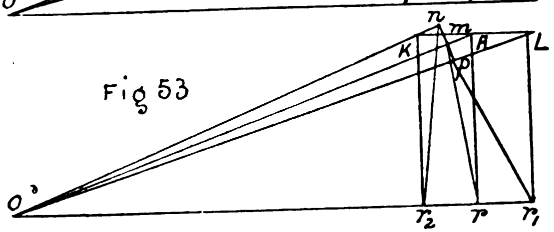


Fig 53

subscript 2 to dimensions at the inner. Then in fig. 52 O r is the mean radius 4·925, O F and O E are 5·6 and 4·25, the outer and inner radii at the bottom of the wheel, B O r is made equal to θ. Then practice would make θ_1 = D O F, θ_2 = C O E. If the arc G H be struck with radius O B, and G E and H F be joined, then

$$\frac{u_2}{\mathrm{GE}} = \frac{u}{\mathrm{B}r} = \frac{u_1}{\mathrm{HF}}$$

as G E the greater is only about 1·2 B r the lost energy will not be more than 1·44 $\frac{u_2}{2g}$ per pound of water, while the work done in the wheel per pound will not differ much from

$\frac{w_1 c}{g}$, because the products $w_2 c$ for the radii which are less than r are negative, while for radii greater than r they are positive and but little more than the negative values.

Fig. 52 gives $\theta_1 = 15° 4'$ and $\theta_2 = 19° 31'$ by construction, and the same may be obtained by calculation. Fig 53 is the construction for the guide vane angles, $r_2 r_1$ being made equal to b. Then, if $r m$ be drawn so that $O r m = \phi$ and the arc $n m p$ have its centre at O, $n r_2$ O, $p r_1$ O are the correct values of $\phi_2 \phi_1$, which in practice would not differ much from $O r m$. It appears, then, that there would be a saving of energy by giving $\phi_1 \phi_2$ their correct values. Although θ_2 could be given its correct theoretical value, θ_1 cannot in this case, for

$$\cos \theta_1 = \frac{c_1}{v_2} = \frac{r_1}{r} \frac{c}{v_2} = \frac{5 \cdot 6}{4 \cdot 925} \times \frac{18 \cdot 35}{19 \cdot 2} = 1 \cdot 085$$

and $\cos \theta_1$ must be less than unity.

$$\cos \theta_2 = \frac{c_2}{v_2} = \frac{r_2}{r} \times \frac{c}{v_2} = \frac{4 \cdot 25}{4 \cdot 925} \times \frac{18 \cdot 35}{19 \cdot 2} = \cdot 825$$

$$\theta_2 = 34° 25'.$$

Although it would be worth while to correct ϕ, it would make very little difference whether θ was corrected or not, as is shown by fig. 52.

We have assumed that, although a and θ vary, v, and consequently v_2, are constant throughout the whole width of passage; for if at one point of a guide passage the velocity were not equal to that at another, both points being in the same horizontal plane, but at different radii from the centre, the pressure would be greater at the point where the velocity was least, and *vice versa*; consequently a cross-flow would be set up from the point of higher to that of lower pressure, which clearly could not long continue, and which, when it ceased, would have made the pressures at the two points equal; but if p be this pressure, and v the velocity,

$$H = \frac{p}{62 \cdot 5} + \frac{v^2}{2g} \text{ neglecting friction}$$

$$v^2 = 2g\left(H - \frac{p}{62 \cdot 5}\right)$$

hence if p is the same for both points, v must be the same, and also v_2, and we cannot be far wrong if we calculate these velocities on the assumption that all the water flows through the wheel with the same velocities as at the mean diameter.

CHAPTER XIII.

CONSTRUCTION OF THE VANES OF TURBINES.

THE following is the method of constructing the guide vanes at the mean radius of a parallel-flow turbine. Take A B (fig. 54) equal to the pitch, and make the angle C A B equal to α; draw B C D perpendicular to A C, cutting the line representing the top of the guide apparatus in D; with centre D and radius D C describe the arc C E; then all the other vanes may be made in the same way as A C E. The water flows downwards at first on entering the guide passages, and as their cross-section decreases gradually increases its velocity. K is unity, because A C and B C are parallel, and therefore there is no contraction. To construct

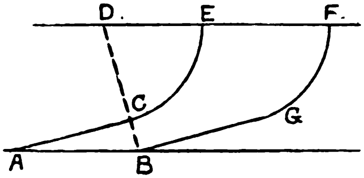


FIG. 54.

the wheel vanes (fig. 55), take A B equal to the pitch, and make B A C equal to θ; draw B C D perpendicular to A C. A point D has now to be found as the centre of the arc C E, so that the angle G E F may be ϕ, E F being tangent at E. Take any line H L so that G H L = ϕ, and make L H K = a right angle; bisect C K H, and draw C E at right angles to this bisector, and the centre D may now most readily be found by trial, or by drawing a perpendicular through the centre of C E.

Absolute path of a particle of water through the wheel.—It is advisable to draw this in order to assure one's self that there are no rapid bends in the path. If there are any sharp bends, there will be a loss of energy, although there is no well-established formula expressing the loss known to the author, and the vane curve must be altered. When the breadth of the wheel is constant, the downward component of the velocity of the water is also constant if we can neglect the thickness of the vanes, which, for the above purpose, we can do safely. It is then easy to draw the absolute path

either by the following graphical method or by calculation. Let O B be the section of a vane at the mean radius; draw $O v_1 A c$ the parallelogram of velocity at entry. Then when the particle of water has moved through vertical distances $O h$, $O n$, $O s$, the wheel has moved through horizontal

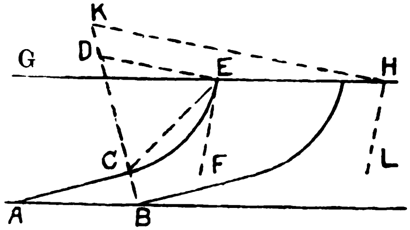


FIG. 55.

distances $f g$, $l m$, $t v$; hence k has moved to q, p to r, B to u, if $k q = f g$, $p r = l m$, and $B u = t v$. $O q r u$ is therefore the path of the water. Of course we need not limit ourselves to three points. If the axial velocity of the water is not constant, we may proceed as follows: Let us suppose two sections taken through the wheel at right angles to the

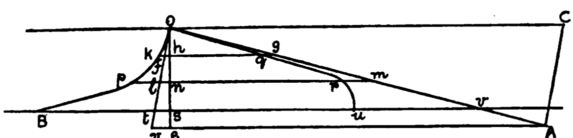


FIG. 56.

axis, the one where the water enters and the other where the water leaves the wheel. Let the first area = B square feet, and the second B_2 square feet. Then $B = \dfrac{a}{\sin \alpha}$, $B_2 = \dfrac{a_2}{\sin \theta}$.

Let B_1 = a similar area at a vertical distance x below the plane containing B. Let us assume, what will be sufficiently accurate, that the increase of area from B to B_2 varies directly with x.

Let h = depth of wheel; then

$$B_1 = B + \frac{x}{h}(B_2 - B) = B + k x,$$

so that

$$k = \frac{B_2 - B}{h};$$

$\therefore$ vertical component of the velocity

$$= v \sin \alpha \frac{B}{B + k x}.$$

Let t = time from B to B_1.

Then $$\frac{d x}{d t} = \frac{l}{B + k x} \text{ where } l = B v \sin \alpha.$$

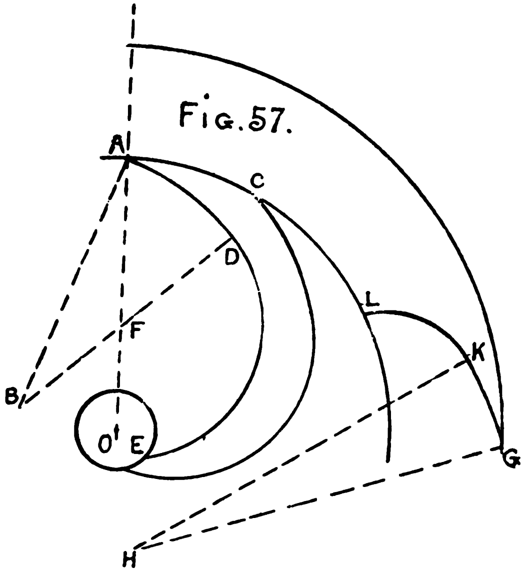
$$\therefore (B + k x)\, d x = l\, d t$$

$$B x + k \frac{x^2}{2} = l t$$

$$B x + \frac{(B_2 - B)\, x^2}{2 h} = B v \sin \alpha . t$$

$$\frac{B x + \frac{(B_2 - B)\, x_2}{2 h}}{B v \sin \alpha} = t.$$

FIG. 57.



During time t, the motion of the wheel at the mean radius is $c t$; hence, in fig. 56, we may take distances O h, O n, O s, and substituting their values in the last equation in place

of x, we may find t, and thence $c\,t$, $f\,q$, $p\,r$, B u, may be made equal to the three values of $c\,t$ thus obtained, and the curve O $q\,r\,u$ traced. In fig. 56 the curvature of the actual path of the water is greatest near the end, which is as it should be, because then the velocity is least. It is probable that the loss caused by curvature increases as some power of the velocity, and is inversely proportional to some power of the radius of curvature of the path of the water.

The following is a method of constructing the curves for the vanes of a Fourneyron turbine. Fig. 57 is a plan of the guide apparatus and the wheel. Let A be a point on the

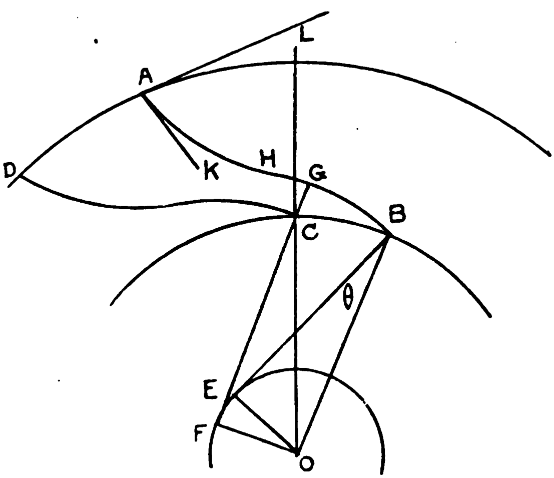


FIG. 58.

circumference of the former, from which a guide vane is to commence. Take A B = A O, and the angle B A O = α, and make the arc A D = arc A C, the pitch of the guide vanes at the circumference. From D take D F = $\frac{1}{2}$ B D, and with radius D F draw the arc D E.

For the wheel vanes take O G H = θ, and G H = O G, and draw the arc K G, making its length equal to the outer pitch of the vanes. Some point on K H must next be found, from which as centre the arc K L may be described, so that the angle between the tangents to the two circles meeting at L may be equal to ϕ.

For an inward-flow turbine, vanes which are in part involutes of a circle are used to lessen the contraction of the stream issuing from the passages. The construction is shown in fig. 58; here O A is the external radius, O B the internal. Take O B E = θ, and O E a perpendicular to B E, and with O E as radius draw a circle. Let B C be the inner pitch, and suppose a thread wound round the circle E, and carrying a pencil at B. As this is unwound the arc B G H is traced by the pencil, the point H being a little to the left

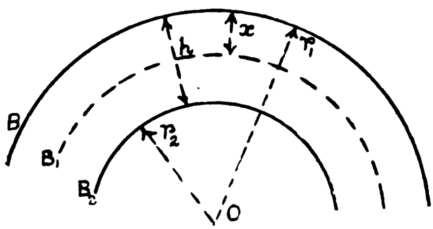


FIG. 59.

of the line F C G, and the width of the passage being therefore constant between H and G. This will lessen, but will not altogether prevent, contraction. The continuation of the vane H A is the arc of a circle, the tangents A K and A L containing the proper angle ϕ. The guide vanes may be constructed in the same manner, except that instead of ϕ we must have an angle of 90 deg., and instead of θ the angle α.

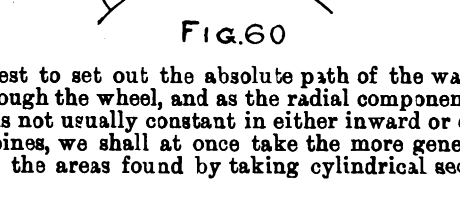


FIG. 60

It is best to set out the absolute path of the water as it flows through the wheel, and as the radial component of the velocity is not usually constant in either inward or outward flow turbines, we shall at once take the more general case, in which the areas found by taking cylindrical sections at

the outer and inner circumferences of the wheel (the axis of these cylinders being the axis of the wheel) are B and B_2. We may assume with sufficient accuracy that if B_1 is an intermediate section,

$$B_1 = B + \frac{x}{h}(B_2 - B),$$

where h is the difference between outer and inner radii, and x is the difference between the outer radius and the radius at which B is taken. We shall only consider the case of inward flow, and the above remarks are intended to apply to this alone.

Fig. 59 shows at a glance the dimensions r_1, r_2, x, h, and the circles corresponding to B, B_1, B_2.

$$B = \frac{a}{\sin \alpha} \quad B_2 = \frac{a_2}{\sin \theta}.$$

It will be noticed that we assume that $\frac{a}{\sin \alpha} = \frac{a_1}{\sin \phi}$ where a_1 is the area at entry into the wheel, and although this is not exactly the case, the difference should be as small as possible to prevent a sudden change of section of the stream. Then, $B_1 = B + k x$, where $k = \frac{B_2 - B}{h}$. The radial component of the velocity is $\frac{d x}{d t} = v \sin \alpha \frac{B}{B_1}$

$$= v \sin \alpha \frac{B}{B + k x} = \frac{l}{B + k x} \text{ where } l = B v \sin \alpha.$$

Integrating, we get $B x + k \frac{x^2}{2} = l t$,

t being the time a particle of water takes in passing from area B to area B_1.

$$\frac{B x + \frac{(B_2 - B) x^2}{2 h}}{B v \sin \alpha} = t.$$

Let n be the angle turned through per second by the wheel. Then $n t$ is the angle turned through in time t, and in fig 60, if any number of points D, E are taken on a vane, and the values of x corresponding to these points be put in the last equation, t, the time from A, may be calculated. During this time the wheel has turned through the angle $n t$, and the arcs D F, E G, and B C may be taken, subtending this angle at the centre. Then A F, G C is the actual path of a particle of water.

CHAPTER XIV.

THE REGULATION OF REACTION TURBINES.

IT is necessary when the power required from a turbine decreases to lessen the quantity of water passing through it, and, speaking generally, this may be done in two ways, by hand or by a governor of the indirect-acting type, about which we shall speak on a future page. Methods of regulation may also be divided again into two classes: firstly, those that close some of the guide passages com-

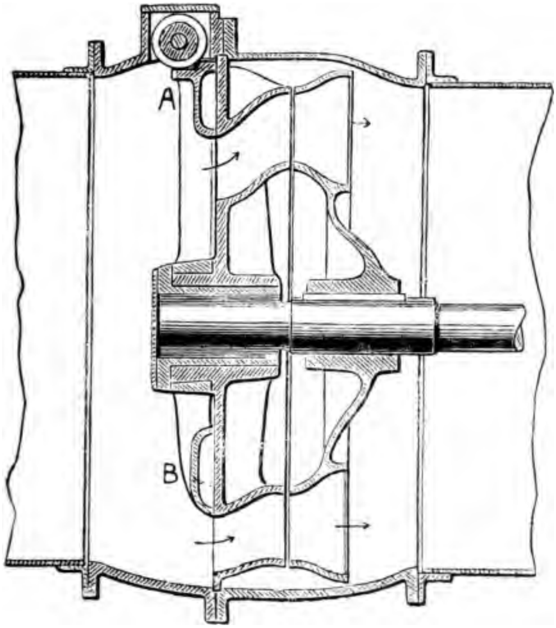


FIG. 61.

pletely, and, secondly, those that close all the guide passages equally; the former are usually economical, and the latter reduce the efficiency considerably the more the

sluice or gate is closed. An exception to this statement is found when the turbine wheel is divided by partitions at right angles to the axis in a radial-flow turbine, when the efficiency is but slightly reduced, as we have already shown in the case of the Hercules turbine, in which an efficiency of 71 per cent was obtained when the sluice opening was ·379 of the whole, while the efficiency was 84 per cent at full gate; this is a very fair result. The method of the late Professor Thompson of altering the inclination of the guide vanes, and thus decreasing the section of the guide passages, has also given excellent results, although the direction of flow is suddenly changed on entry to the wheel. This is probably due to the fact that the alteration a is not sufficient to seriously affect the efficiency.

Figs. 61, 62, 63 show three views of an axial turbine, with horizontal shaft, constructed by Mr. W. Günther, Central Works, Oldham. It is of 300 horse power, with a fall of 57 ft., and the end thrust is taken by a collar bearing, not shown in the figures, and it drives a rope pulley direct. The bearings inside the casing are made of lignum-vitæ strips, so that they may be lubricated by water, and they are protected from grit. The mean diameter of the wheel is 3 ft. 6 in., and it makes 200 revolutions per minute, and the vanes are made of steel bent to template, and placed in the mould before casting. This lightens and strengthens the wheel, and the efficiency is greater than with cast-iron vanes. The supply pipes are 42 in. diameter, and the turbine is placed 15 ft. above the tail race. The sluice is a slide, the two parts of which are marked A, B, figs. 61, 62, and it is seen in section on the top of fig. 63. The guide passages (fig. 62), some of which are closed, are in two semi-circular divisions, so that the outer division can be closed by A, and the inner by B. Passages are thus shut off equally on either side of the centre. This is not always the case in the regulation of turbines, although it undoubtedly should be, for the action of the water in this case produces a couple, and the only pressure on the lignum-vitæ bearings is that due to the weight of the turbine; otherwise we should have in addition a force equal to the resultant of the pressure on the wheel vanes, but opposite in direction, this force being exerted by the bearings, and, consequently, the friction thereat would be greater. The slide is rotated by a worm and semi-worm wheel, actuated by a hand wheel and bevel gearing. The reason that such an arrangement as the above is efficient is that part of the passages are entirely closed, and the remainder entirely

open, so that the flow of water through these is not interfered with, and is exactly as calculation has arranged that it should be. There are, however, two possible, and it would seem unavoidable, causes of loss of efficiency. When a passage is only half closed, there is a sudden enlargement after passing the slide, and when a vane such as C leaves the open passage D (fig. 63), water still flows into it from

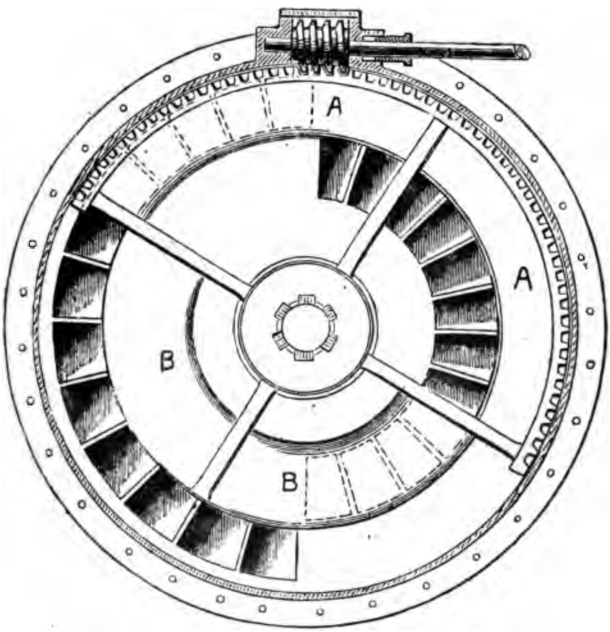


FIG. 62.

D until the next vane E takes the place that C now occupies, and so the water from D is not deflected by the upper part of the vane C, as it should be, but strikes C lower down, and the consequent sudden change of direction causes a loss of energy.

If the work required were suddenly decreased, a throttle valve in the suction pipe can immediately lessen the flow. This may also be used for rapidly starting or stopping the

wheel. This type of wheel is suitable for falls up to about 60 ft. or 70 ft. For the above purpose a throttle valve is an excellent arrangement, but if it is the only regulator it decreases the rapidity of flow through all the guide passages; the water enters the wheel with shock, and does not leave it axially, but with a positive velocity of whirl. The work done per pound of water and the efficiency are reduced in consequence. The resistance of a throttle valve alone in a stream would set up eddies, and thereby waste power. It is sometimes, however, used in the head race or suction tube.

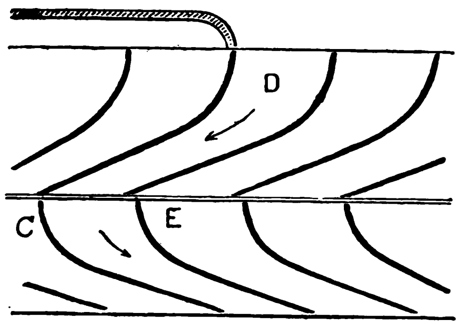


FIG. 63.

Another economical arrangement, in which each passage is either completely open or closed, is shown in figs. 64, 65. There is a slide *a* to each guide passage, three of these being connected to a crossbar *c*. The whole is carried by the central rod, which, by means of a horizontal pin and roller, is supported in one of the horizontal grooves (fig. 65) of a cam, which is turned by means of a hand wheel and bevel wheel. As it rotates, the inclined path is descended by each of the rollers, the three slides thus falling and closing the passages. This is a more cumbersome arrangement than the last, and unless there are two inclined paths on the cam the closing of the passages will not be equal on opposite sides, with the disadvantage explained above. Flaps closing the passages are also used, the hinges being at right angles to a radius through their centres.

Fig. 66 shows a method of regulation by conical rollers P P, and annular guttapercha strips, whose ends are fastened—two to the guide apparatus, and the other two to the rollers. These latter can turn not only about their geometrical

inclined axes, but also about the axis of the turbine, so as to wind the guttapercha strips upon themselves when the passages are to be opened, and the reverse when they are to be closed. For this purpose the vertical shaft A carries a pinion B, which engages with the tooth sector C, on which

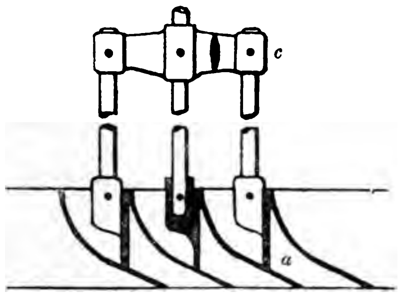


FIG. 64.

are fastened the arms D, which with their forked ends take hold of the rollers. In order to strengthen the guttapercha strips against the water pressure, a number of iron plates are placed close together, and across the guttapercha. Leather is better than guttapercha.

There is another method which is ingenious, but we think hardly so good as those mentioned above. There is a small

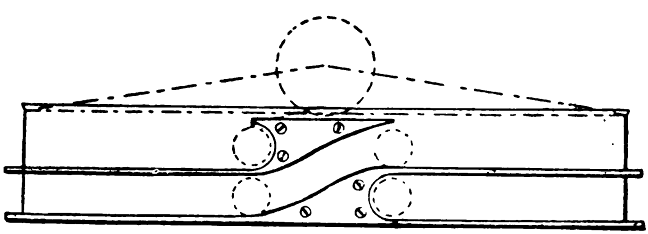


FIG. 65.

pinion, as in fig. 66, which drives a large spur wheel, whose axis coincides with that of the turbine. This wheel carries a long arm, projecting radially, which, as it moves round, strikes in turn the arms of a number of two-armed levers, whose vertical pivots are placed on a circle, just inside the

guide passages. These two arms are at right angles, and as the spur-wheel arm moves to the left, it pushes the left arm of the lever to the left and outwards, while the other arm moves inward. Upon this second arm is a pin, to which is connected one end of the connecting link, the other end of which is attached to a horizontal slide, which moves radially inwards when the above action takes place, and closes several guide passages. To open the guide passages the spur-wheel arm moves to the right, and pushes back the second arm of the lever, which thus moves the pin it carries outwards, and by means of the connecting link pushes back the slide. The principal disadvantage of this arrangement is that all the

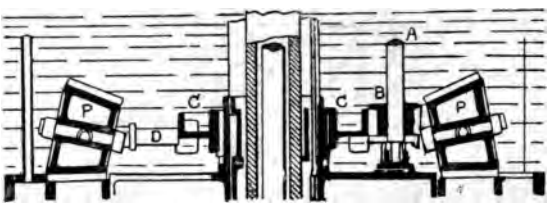


FIG 66.

working parts are under water, and the action of grit and dirt on such a large number of pins would cause a considerable amount of wear.

As an illustration of the advantage of completely closing some of the guide passages instead of throttling them all equally, we give the following example from "Zeitschrift des Vereins Deutscher Ingenieure" of a turbine regulated by flaps, as explained above: Its efficiency at full gate varied between ·8 and ·83, while at three-quarter gate it was from ·78 to ·79, which only fell to about ·75 when half the passages were open.

We have already spoken of the sub-division of axial and radial turbines; and for the regulation by a sluice at the bottom of the suction tube, or between guide passages and wheel, which latter is moved parallel to the axis, we must refer the reader to a former page.

It will be remembered that we gave a description of the Victor mixed-flow turbine, but that we were unable to give a drawing of a guide apparatus and sluice. We have now received from Mr. Frederic Nell several drawings of the parts of the turbine. Fig. 67 shows the guide apparatus, or "outer chute case." About the centre of the figure may be

seen the wood step upon which the shaft of the wheel is supported. This cylinder is in one casting, and it is bored out to receive the sluice or register gate, which in this case forms part of the guide apparatus, fig. 68, containing as it does half the length of each guide vane. These taper from the middle to either end, so that when the gate is open the water flows freely to every part of the wheel, and this sluice is therefore superior to that shown in fig. 45. It is bored out to receive the wheel, and turned to fit into the outer case, within which it revolves and is moved, for the purpose of admitting and shutting off water by means of the segment

FIG. 67.

at the top of fig. 68, and a pinion which gears with it. We have explained above that it is impossible for this or any other arrangement that throttles the flow to give a good efficiency at part gate, and the experiments at Keswick, above mentioned, show this, although at full gate this is an excellent turbine, and, according to the experiments at the Holyoke testing flume, has given efficiencies between ·8289 and ·896, with heads from 11·65 ft. to 18·34 ft. At part gate there is a sudden contraction at the sluice, and a subsequent enlargement of the passage again. As water flows in less quantities to the wheel, it cannot enter with the proper velocity that would prevent shock, nor will it flow out axially under these circumstances. Fig. 67 is a view from

the top, and fig. 68 from the bottom. Fig. 69 is a view from underneath of the top of the wheel case, which is bolted to the latter, fig. 70. It is composed of a single casting, with a pedestal attached, and protects the wheel from the vertical pressure of the column of water. The projection of the

FIG. 68.

pedestal underneath the top fits into the ring or hub of the "gate spider," this being the name given to the four arms at the top of the sluice, two of which are visible in fig. 68. The pinion and segment are also enclosed in the casing, but can be readily got at by removing the cap, fig. 70, at the left of the top of the wheel case. The top shown in fig. 70 is of

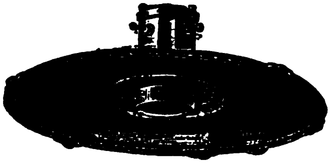


FIG. 69.

later design than that in fig. 69, the pedestal being reduced in height, so that the wheel may be exposed to view by lifting the outer chute case, gate, and top without removing the coupling from the main shaft. Fig. 70 shows the wheel complete, and fig. 71 a pair of wheels with case and governor, which latter we shall describe later.

Fig. 72 shows a curious method of regulation, in which the guide vanes are, as it were, split in two halves, of which the right-hand part is fixed and the left movable;

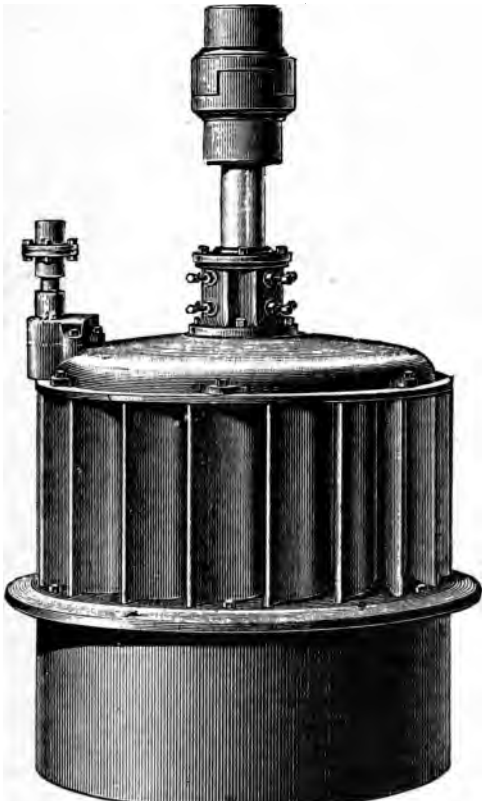


Fig. 70.

this latter is suspended by the rod B from a ring A, which can be raised or lowered. C C are guides, to cause the movable halves to descend vertically. When completely

lowered, they occupy the position shown by the dotted curve D. Although this method throttles the flow in each passage, it gives a fair efficiency, as shown by the following table for a 60 in. Collins axial turbine.

Gate opening.	Head.	Cub. ft. per sec.	Horse power.	Efficiency.
1·000	16·56	64·88	102·18	84·01
·548	17	50·92	69·68	71·10
·297	17·53	34·53	36·76	53·64

Fig. 73, from Rankine's "Steam Engine," shows an old-fashioned arrangement, which is somewhat similar in principle to the above, but which was not efficient, and has therefore given way to some of the better plans mentioned above.

CHAPTER XV.

TURBINE GOVERNORS.

ALL water-wheel governors are of the indirect-acting class—that is, they enable the wheel to close or open the sluice by its own power, the governor itself not being sufficiently powerful to perform the work directly. We give here two views of the Snow governor, as applied to the Victor turbine. The first, fig. 74, is a perspective view, and the second, fig. 75, is an outline drawing showing the principle of its action. In fig. 74 will be seen two shafts, vertical and horizontal; the former turns the pinion which gears with the spur segment on the sluice, and the latter drives the governor. Upon this horizontal shaft at its right end is a pulley, not shown in the figure, which is driven by belt from the turbine shaft, while at the left end there is a pinion which is concealed by the spur wheel at the extreme left of the figure, which spur wheel the pinion drives, and this, by means of two bevel wheels, drives the governor balls. The arms of the governor have teeth on their inner ends, which are in gear with the central spindle, so that according as the balls fall or rise the spindle rises or falls. Fig. 75 shows the connection between this spindle and the pawl shifter. B C D is a lever, with fulcrum at C; D G is a link connecting it with the sector, which is called the pawl shifter, and which is pivoted at H. Returning to fig. 74, we see just at the right of the base of the governor a small crank upon the same shaft as the spur wheel at the left of the figure, which by its rotation gives a reciprocating

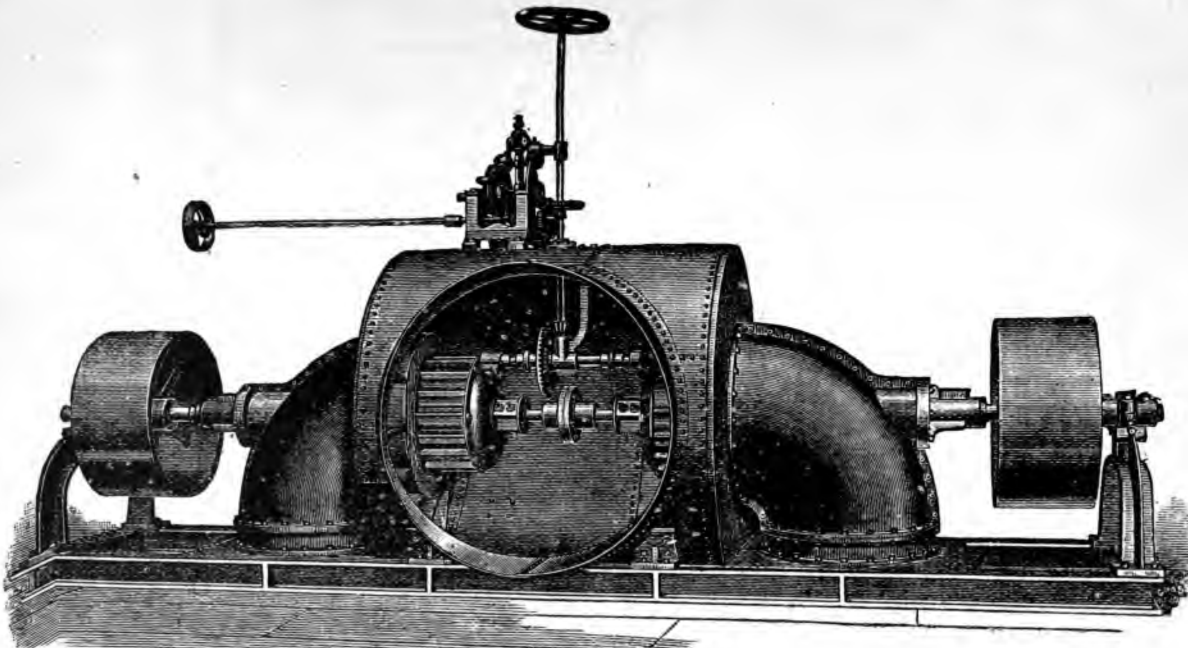


FIG. 71.

motion to the short connecting rod, at the upper end of which are two pawls, called the hoisting and closing pawls, because the former opens and the latter closes the sluice when they are allowed to gear with the ratchet wheel, which lies just at the left of the upper end of the connecting rod. The sector, or pawl shifter, fig. 75, part of which is just visible to the left of the ratchet wheel in fig. 74, would, if its upper edge were circular, prevent the pawls gearing with the ratchet wheel; but the depression at the centre of the circumference, fig. 75, enables them to fall into gear when required. For, if the balls rise B is lowered, and D

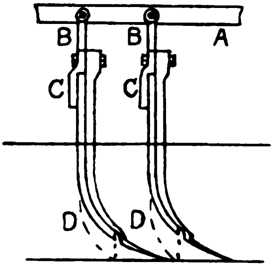


FIG. 72.

and G are raised, so that the depression on the pawl shifter allows the closing pawl to gear with the ratchet wheel. If the balls fall, motion in a reverse direction takes place, and the hoisting pawl gears with the ratchet wheel. This ratchet wheel turns the vertical shaft by means of the two bevel wheels shown at the right of fig. 74, the larger of which is on the vertical shaft. In order to prevent over-winding—*i.e.*, to prevent the governor from continuing to move the sluice when it is full open, and when the wheel is running below its proper speed, when the water is low in the river or stream, there is an arrangement of reducing gear between the ratchet wheel and bevel pinion, which brings into position a stop similar to the sector, or pawl shifter, which will only allow the pawls to slide freely along its top, and prevents them engaging with the ratchet wheel, and thus prevents further motion of the sluice. In order to regulate the speed, there is a horizontal bar on the pawl shifter, upon which slides a weight *k*, fig. 75; if this is pushed to the right, it will tend to raise D and lower B and

lift the balls of the governor, so that in order that the pawl shifter may be in its middle position, with neither pawl in gear, the speed of the wheel must be less. For a similar reason, if it is necessary to increase the speed, k should be shifted to the left. This governor is said to be very rapid in its action.

Hett's governor is shown in fig. 76. Its action depends mainly on the differential motion of three bevel wheels. We shall not attempt to explain this motion, as it should be

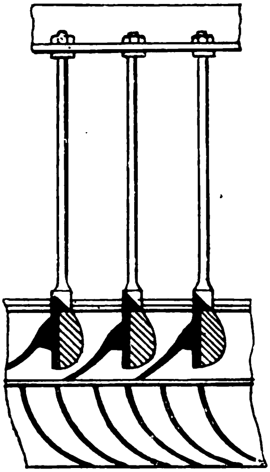


FIG. 73.

understood by most of our readers, who may refer to Prof. Goodeve's "Elements of Mechanism," if they are ignorant of the subject. In this governor the initiative is given by a governor of the Porter type A, fitted with heavier balls than usual. As this rises or falls it moves a strap shifter B, which changes the position of the crossed belt C on the tapered cones E and F. Thus, when the governor sleeve is in mid-position, the belt is on the centre of the cones and E is driven at the same speed, but in the opposite direction to F, which receives its motion from the pulley G. When the sleeve falls the belt is shifted to the left, so that E is driven faster than F, and when the sleeve rises the reverse takes place. The lower cone shaft drives the governor

spindle by means of mitre bevel wheels. The upper cone shaft drives a sleeve H D, which is loose on the governor spindle, and carries the upper bevel wheel of the differential motion D at its lower end. The lower pinion of D, being attached to the governor spindle, revolves with it. As the gearing is the same, it is clear that when the governor is running at its proper speed, so that the belt is on the

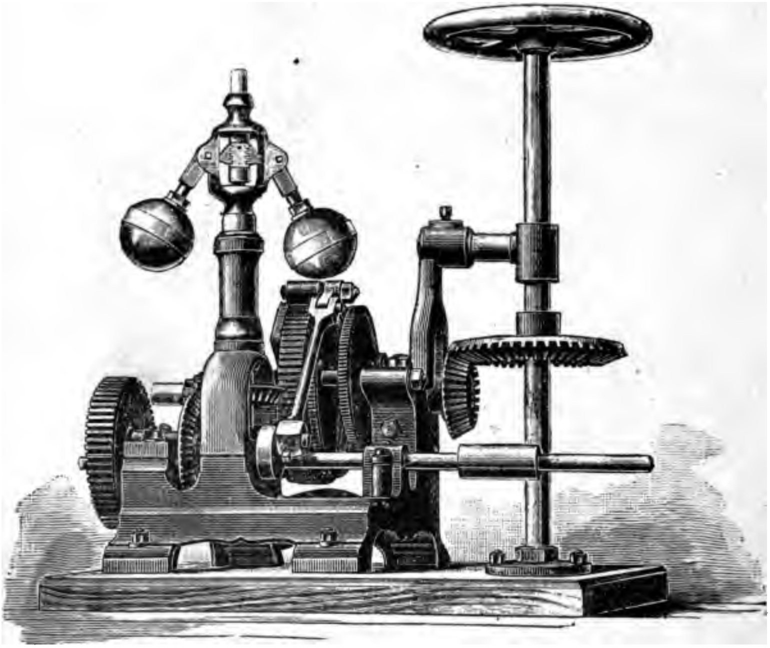


FIG. 74.

middle of the cones, the upper and lower bevel wheels of the differential motion run at the same speed in opposite directions, and the two other bevel wheels in the cup K rotate on their axis without giving any motion to the cup K. When the upper bevel wheel runs at a different speed to that of the lower one, the cup K rotates and moves the

pinion at its base, which gears with the spur wheel on the hand-wheel spindle of the starting gear.

To connect the motion of the hand-wheel spindle a spring clutch is employed, which can be instantly put in or out of gear by giving a quarter turn to the disc, shown immediately

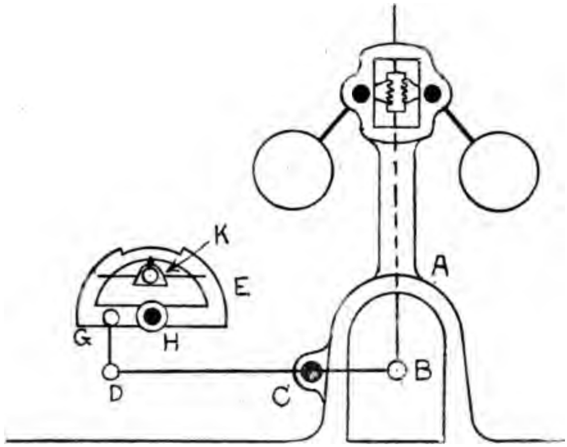


FIG 75.

above the spur wheel. The clutches are thrown out when stopping by hand, and will slip if the running speed is not reached when the gate is full open. One great advantage of this arrangement is that the speed of regulation increases in proportion to the amount of irregularity to be corrected.

CHAPTER XVI.

THEORY OF IMPULSE TURBINES.

THE two principal types of impulse turbines are the axial, with full admission—*i.e.*, admission all round when working at full power—and the radial outward flow, with only partial admission; the former having vertical shafts and the latter horizontal. Impulse turbines are much used in Europe, where the water supplies are variable, as they permit of regulation without loss of efficiency, which, it has been

shown, is not the case with reaction wheels. Also, where the fall is great compared with the quantity of water, the latter becomes very small; on the other hand, the former cannot be used with a suction tube, and therefore the wheel must be placed close to the tail-race level, or there will be a loss of head. This entails a longer shaft, and if variations occur in the level of the tail race, the impulse wheel will then work drowned, and with a diminished efficiency in consequence. It is, indeed, possible to design a wheel so that

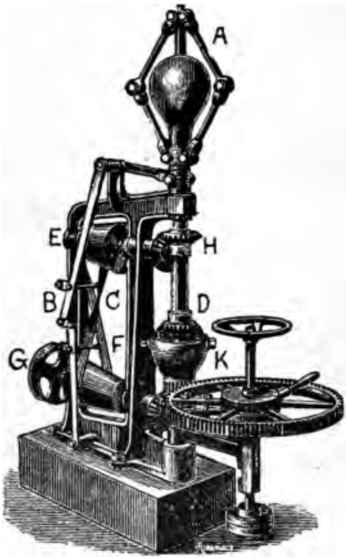


FIG. 76.

it will work both as an impulse and reaction turbine, but it will not be so efficient as one designed for special conditions. Thus reaction wheels will be used where there is a constant supply of water which is not small compared with the head, and where the power required is not very variable. Impulse wheels should be used for high falls and comparatively small quantities of water; also, if the supply is variable, these will be preferable; while a turbine designed for both reaction and impulse may be used where the fall

and supply are suitable for both types of wheel, but a rise in the tail race occurs simultaneously with a reduction in head and an increased supply of water. Thus for two or three months in the year the supply might be 130 cubic feet per second, with a fall of 10 ft., which in drier seasons might be altered to 100 cubic feet and 12 ft. fall, when one of the above type would be suitable. The water in impulse turbines has free deviation, and its pressure is atmospheric. Hence the law of continuity, $Q = a\,v = a_2\,v_2$, does not hold good, and although the pressure on the wheel vanes is caused by change of momentum, just as in the case of the reaction wheels, the equations that we must use for design are different.

Axial Impulse Turbines.

We shall first consider the above type. As before, we assume that the outflow is axial, and hence we obtain—

$$\text{Work done per pound} = \frac{c\,w_1}{g}$$

$$= \frac{c\,v\cos\alpha}{g}.$$

If it were not for friction, v would be equal to $\sqrt{2\,g\,h_1}$, where h_1 is the head above the guide passage orifices; but we must use the equation—

$$v^2\,(1 + F) = 2\,g\,h_1,$$

where F is a coefficient of resistance lying between ·23 and ·02. Taking a mean value ·125—

$$v = 8\sqrt{\frac{h_1}{1\cdot125}}$$

$$v = \cdot75\sqrt{h_1} = \cdot94\sqrt{2\,g\,h_1} \quad . \quad . \quad . \quad . \quad (27)$$

We can now get a clearer idea as to the mean radius of the wheel, and from this correct h_1, which we at first assume as 1 ft. less than H, the total head.

The depth of the wheel may be from one-eighth to one-eleventh of the mean diameter, and, in reality, experience is our best guide. The radius, if calculated, can be obtained in the following manner:—

The area of the guide passages is a, and theoretically we should expect

$$a\,v = Q;$$

but in practice we find

$$a = \text{about } \frac{9}{8}\frac{Q}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

also $$r = k\sqrt{a},$$

where k varies between 1·25 and 2.

Having now chosen some value for the depth of the wheel, which we shall call h_2, and for h_3 the clearance of the wheel above the tail race, we can correct h_1, a, and v; but the alteration will probably not be sufficient to require any alteration of r.

It is convenient, for the first approximation at least, to assume that the work done by the wheel per pound of water is $\frac{v^2}{2g}$, while the remaining head h_2 is used to overcome the friction of the wheel, and supply the necessary velocity of flow from the wheel. Assuming, therefore, that

$$\frac{v^2}{2g} = \frac{c\,w_1}{g} = \frac{c\,v\cos\alpha}{g},$$

then

$$c = \frac{v_1}{2\cos\alpha}.$$

Fig. 77 shows that the above requires

$$c = v_1 \text{ and } 180 - \phi = 2\,\alpha.$$

Thus the angle $v_1\,O\,c = 2\,v\,O\,c$.

After entering the wheel with a relative velocity v_1, the water falls a distance h_2, during which fall its relative velocity become v_2; so that, if F_2 is the coefficient of resistance of the wheel referred to v_2,

$$v_1^2 + 2\,g\,h_2 = (1 + F_2)\,v_2^2 \quad . \quad . \quad . \quad (29)$$

$$v_2 = \sqrt{\frac{v_1^2 + 2\,g\,h_2}{1 + F_2}}$$

$$\cos\theta = \frac{v}{2\cos\alpha}\sqrt{\frac{1 + F_2}{v_1^2 + 2\,g\,h_2}} \quad . \quad . \quad . \quad (30)$$

whence we may obtain θ. Perhaps (30) will be more evident if we remember that for axial outflow

$$v_2 = c\sec\theta = \frac{v}{2\cos\alpha\cos\theta}.$$

F_2 may be taken as ·1.

For medium quantities of water, from about 30 to 60 cubic feet per second, with falls from 25 ft. to 40 ft., α should lie between 15 deg. and 18 deg., a small angle only being required; while θ may be between 13 deg. and 16 deg. For larger quantities of water, from 40 cubic feet to about 200 cubic feet, with falls from 5 ft. to 30 ft., α may lie between 18 deg. and 24 deg., and θ between 16 deg. and 24 deg. For larger quantities of water and lower falls, α may be from 24 deg. to 30 deg., and θ from 24 deg. to 28 deg.; the reason for larger values of α and θ with lower falls and larger quantities of water is, that the increase of these angles will increase the quantity of water flowing through the wheel, other things being unaltered, while the decrease of head will decrease the flow. Hence if the wheel is not to be made unreasonably

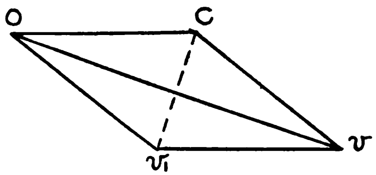


FIG. 77.

large, α and θ must be increased. Again, with increased head and decreased quantity of water, we can increase the value of w, the velocity of whirl, by decreasing α (for $w = v \cos \alpha$), and this can be done without making the wheel unnecessarily large. If we do not get a value of θ agreeing with the above, we must alter ϕ. The effect of increasing ϕ is to decrease the speed of the wheel, and therefore the work that is done, since w_1 remains unchanged; while to decrease ϕ has the reverse effect. Generally, if equation (30) gives too large a value of θ, then ϕ should be decreased, and increased if θ is too small, or if $\cos \theta$ from (30) becomes greater than unity. The reason for this is, that by increasing or decreasing ϕ we increase or decrease v_1; therefore v_2, by equation (29), is increased or decreased; but increasing or decreasing v_1 decreases or increases c, so that with v_2 increased, and c decreased, θ is increased, because

$$\frac{c}{v_2} = \cos \theta,$$

and for the same reason v_2 less and c greater makes $\cos \theta$ increase and θ diminish.

To illustrate the above, let us take a numerical example.

To design a parallel-flow impulse turbine for a fall H = 9·5 ft. and 170 cubic feet of water per second = Q.

$$h_1 = \text{H} - 1 = 8{\cdot}5\,;\; v = {\cdot}94 \times 8\sqrt{h_1} = 21{\cdot}9$$

$$a = \frac{\text{Q}}{v} \times \frac{9}{8} = \frac{170}{21{\cdot}9} \times \frac{9}{8} = 8{\cdot}72$$

$$r = k\sqrt{a} = 1{\cdot}42\sqrt{8{\cdot}72} = 4{\cdot}2 \text{ nearly.}$$

h_2 may still be 1 ft., but ·9 ft. will be sufficient, so that

$$4{\cdot}66\,h_2 = r$$

h_1 corrected from this = H − ·9 = 8·6.

$$v = {\cdot}94 \times 8\sqrt{8{\cdot}6} = 22,$$

$$a = \frac{170}{22} \times \frac{9}{8} = 8{\cdot}7.$$

r and h_2 may be left as above.

$$\cos\theta = \frac{v}{2\cos\alpha}\sqrt{\frac{1 + \text{F}_2}{v_1^{\,2} + 2gh_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

also $$v_1 = \tfrac{1}{2}\,v\sec\alpha,$$

and F_2 lies between ·05, and we shall take it as ·1.

$$\cos\theta = \tfrac{1}{2}\,v\sec\alpha\sqrt{\frac{1{\cdot}1}{\frac{1}{4}v^2\sec^2\alpha + 2gh_2}}$$

Assume α = 25 deg., and all the other quantities being known, except cos θ, we obtain

$$\cos\theta = {\cdot}895 \text{ nearly}\,;\; \theta = 26\tfrac{1}{2} \text{ deg. nearly.}$$

As this does not lie between 16 deg. and 24 deg., we must assume a smaller value of ϕ, and calculate θ from the formula

$$\cos\theta = c\sqrt{\frac{1 + \text{F}_2}{v_1^{\,2} + 2gh_2}} \quad . \quad . \quad . \quad . \quad \text{derived from (29)}$$

$$\cos\theta = \frac{v\sin(\alpha + \phi)}{\sin\phi}\sqrt{\frac{1 + \text{F}_2}{\left(\frac{v\sin\alpha}{\sin\phi}\right)^2 + 2gh_2}} \quad . \quad . \quad . \qquad (31)$$

For ϕ = 127 deg. this gives θ = 13 deg.
ϕ = 128 deg. „ θ = 18 deg.
ϕ = 129 deg. „ θ = 23 deg.

so that ϕ may lie between 128 deg. and 129 deg., and θ between 18 deg. and 23 deg. We shall take

$$\theta = 23 \text{ deg.},\; \phi = 129 \text{ deg.}$$

It may be asked why θ should not be assumed and ϕ calculated from the equation given above. This equation, however, is anything but simple in form when we attempt to obtain ϕ from θ, and the above method of approximation is far simpler. The final calculation of θ is of very little importance, and if ϕ remained 130 and θ were made anything from 20 deg. to 24 deg., the difference in the efficiency of the wheel would be of no practical importance.

The hydraulic efficiency is $\eta = \dfrac{c\,w_1}{g\,\mathrm{H}}$

$$= \frac{v^2 \sin(\alpha + \phi)\cos\alpha}{g\,\mathrm{H}\sin\phi} = \frac{22^2 \times \cdot 438 \times \cdot 906}{32{\cdot}2 \times 9{\cdot}5 \times \cdot 777} = \cdot 814.$$

The actual efficiency would be about ·03 to ·04 less than this, say ·77 to ·78.

It will be evident to those who prefer graphical methods, that they can be used here frequently. This equation (30) may be written—

$$\cos\theta = c\sqrt{\frac{1 + \mathrm{F}_2}{v_1{}^2 + 2\,g\,h_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30\text{G})$$

and c and v_1 can be found graphically, remembering that $2\,\alpha = 180$ deg. $-\ \phi$; again, (31) is the same as the above, while ϕ differs slightly from the above value, and c and v_1 can be again found graphically; also w_1 can be measured off a drawing, and so θ may be found without the use of trigonometry.

We next have to find the quantities b, b_1, b_2, the widths of guide apparatus, and of the wheel at inlet and outlet. Theoretically, there should be more guide vanes than wheel vanes, as the wheel should not be filled at inflow. This rule is not always adhered to in practice, although it undoubtedly should be. The guide vanes are generally of wrought iron or steel, from about $\frac{1}{8}$ in. to $\frac{1}{4}$ in. in thickness. They are placed in the mould of the guide wheel before casting. The wheel vanes are generally of cast iron, tapered at the ends. In the following formulæ, t_1, t_2 may be from $\frac{1}{4}$ in. to $\frac{1}{2}$ in. Then—

$$b = \frac{a}{2\,\pi\,r\sin\alpha - n\,t - n_1\,t_1\,\dfrac{\sin\alpha}{\sin\phi}} \quad . \quad . \quad . \quad . \quad (24\text{A})$$

Let $t = \cdot 2 \text{ in.} = \frac{\cdot 2}{12} \text{ ft.}\,; \; n = 66$

$t_1 = \cdot 4 \text{ in.} = \frac{\cdot 4}{12} \text{ ft.}\,; \; n_1 = 62$

$$b = \frac{8 \cdot 7}{2\,\pi \times 4 \cdot 2 \times \cdot 4226 - 66 \times \frac{\cdot 2}{12} - 62 \times \frac{\cdot 4}{12} \times \frac{\cdot 4226}{\cdot 7771}}$$

$= \cdot 976 \text{ ft.} = 11 \cdot 7 \text{ in.}$

$b_1 = b + \frac{1}{2} \text{ in.} = 12 \cdot 2$

$a_2 = \mathrm{K}_2 \frac{\mathrm{Q}}{v_2}$, where K_2 lies between 1·3 and 2.

This equation merely means that the passages are not filled at outflow from the wheel.

$$b_2 = \frac{a_2}{2\,\pi\,r \sin\theta - n_1\,t_2}$$

$$= \frac{\mathrm{K}_2 \frac{\mathrm{Q}}{v_2}}{2\,\pi\,r \sin\theta - n_1\,t_2}$$

where $\mathrm{Q} = 176$, $v_2 = \frac{v}{2 \cos\alpha \cos\theta} = \frac{11}{\cdot 92 \times \cdot 906}$

$\sin\theta = \cdot 3907$, and let $\mathrm{K}_2 = 1 \cdot 41$, then from the last equation $b_2 = 27\frac{1}{2}$ in.

CHAPTER XVII.

TURBINES AT ASSLING, CARINTHIA.

AT Assling, Carinthia, there are three large axial impulse turbines, lately constructed by Messrs. Ganz and Co., of Buda Pesth. Their power is used for a steel wire and nail mill, and is obtained from the river Sava, being distributed in the following manner :—

Turbine.	Head.	Cub. ft. per sec.	Revolutions per minute.	H.P.
No. 1	76 ft.	118	134	772
No. 2	79 6 ft.	121	137	827
No. 3	84 ft.	124	140	894

the efficiency in all three cases being between 75 and 76 per cent. The power race that carries the water to the pipes above the wheels is of wood, upon stone piers, except where it adjoins the wheelhouses. The sections of the race are such that the velocity in it is a little over 3¼ ft. per second. The power from all the turbines is transmitted by bevel gearing. The water from the power race B (fig. 78) is admitted by wooden sliding gates, lifted by racks and pinions, which latter are driven by worm wheels, controlled

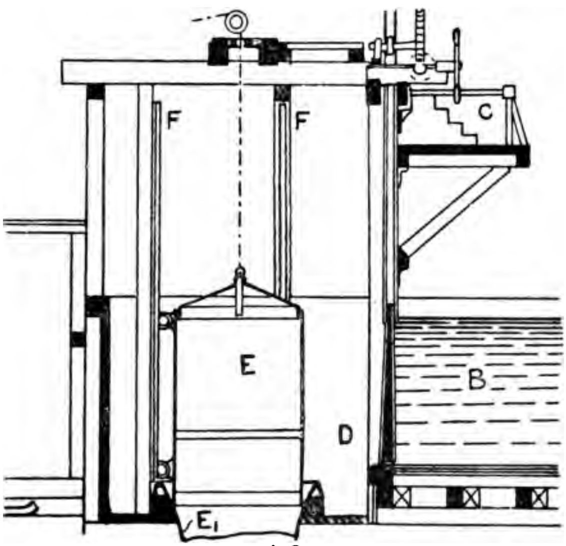


FIG. 78.

by worms and hand wheels on the gangway C. These, however, are too heavy to be used when the wheels require to be quickly stopped and started, which is here frequently necessary for the rolling mills. A ring sluice E is employed for this purpose; in each case it is a sheet-iron sluice of about 5·8 ft. in diameter, with a turned ring at its base, which fits the funnel neck of the pressure pipe E_1. The sluice is guided vertically by rollers and T iron bars F, and is lifted by a chain with a counterweight, passing over a pulley, so

that it may be operated by hand from a suitable position in the mill. The pressure pipe tapers from about 4·92 ft. to 3·93 ft. just above the turbine casing, these pipes being made of cast iron where curved and of wrought iron when straight. As the main points of all three wheels are the same, we shall confine ourselves to the description of No. 1. The turbine casing G, fig. 79, is cast in two parts, with a stuffing box above, and is carried by a ring G_1, fig. 80, which is supported by double tee joists, resting at their centres on pillars K, and

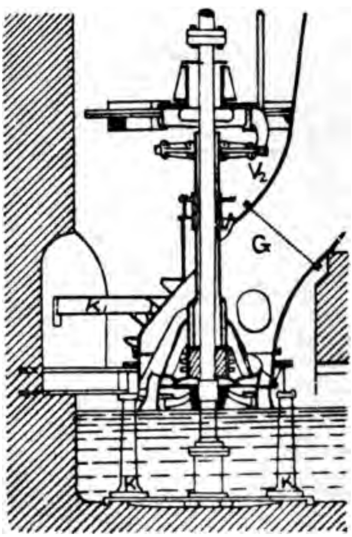


FIG. 79.

embedded in concrete at their ends. There are also two radiating struts, of which one is visible (K, fig. 79), to take the end thrust that comes on the left side of the casing.

The wheel (fig. 80) is 4·92 ft. in external diameter at the top, but is larger at the base, while the internal diameters are the same at both. This is a good form of wheel, for it allows for the tendency of a particle of water to move in a vertical plane after issuing from the guide passages; this carries it away from the centre, and therefore the widening of the wheel, unsymmetrically with regard to the line $J J_1$,

is an advantage. Fig. 81 is a cylindrical section through guide and wheel vanes, the former being alternately of cast and wrought iron, while the latter are of cast iron, and have back vanes, which are of no value when the tail race is

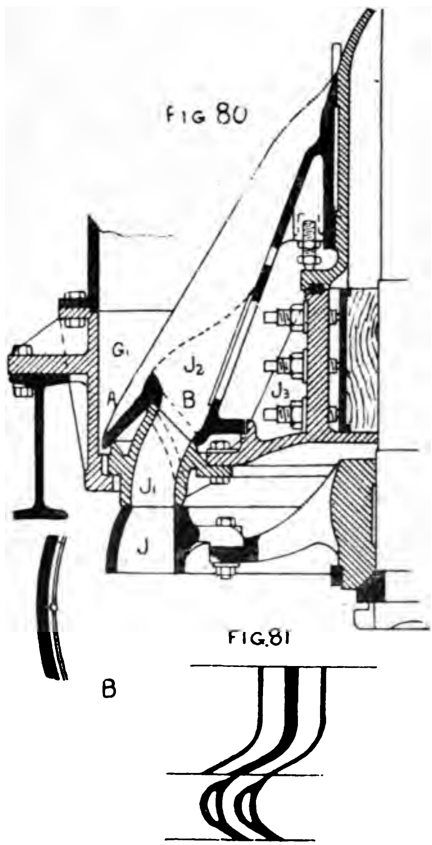


FIG 80

FIG.81

below the wheel; but in times of flood, when the turbine is more or less drowned, and the wheel consequently full of water, they prevent sudden enlargements of the wheel

passages. Back vanes are now only used when the turbine must work at one time as an impulse wheel and at another as a reaction wheel, as they are clearly unnecessary when ϕ is less than a right angle, which is always the case in modern

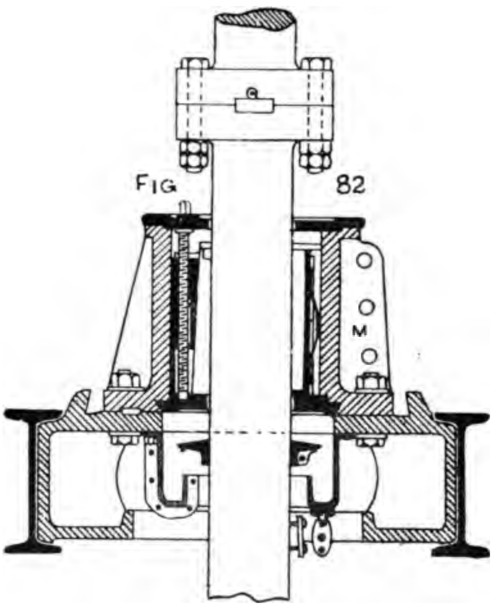
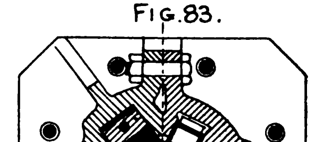


FIG 82

FIG. 83.



reaction wheels. The method of regulation is similar to that of the Günther axial turbine (figs. 61, 62, 63); in this case, however, the faces of the slides and upper surfaces of the guide wheel are not semi-circular rings, but semi-frustra

of cones. Figs. 79 and 80 show a section through the left-hand half of the guide apparatus J_1, and the slide J_2, which is intended to cover the other half of the guide passages, its

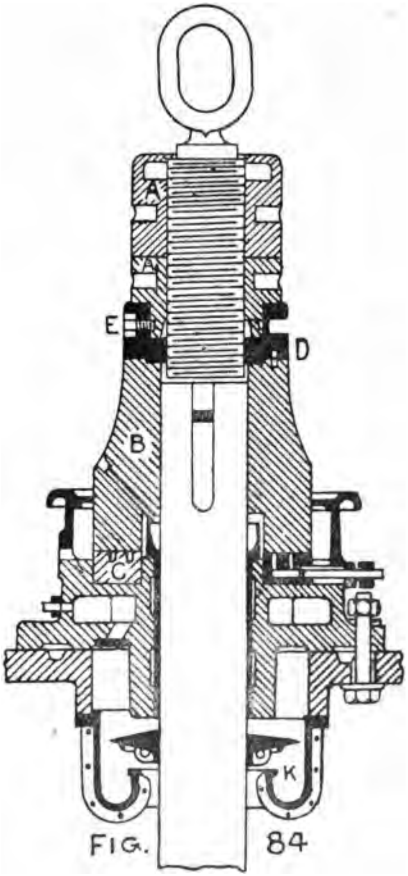


FIG. 84

section being shown at A, while at B the dotted lines show the section of the slide that can close J_1. The upper surfaces of the two halves of the guide ring will clearly meet in a

right angle. The position of the slide is regulated by worm and wheel V_2 (fig. 79), which may be actuated by a governor or by hand gear from above. The shaft is in three lengths, connected by flange couplings. There are five collar bearings, of which the lower one, immediately above the guide ring, is bushed with hard wood, as shown at J_3 (fig. 80); while the other have three metal chucks, adjustable by springs and cottars (figs. 82, 83).

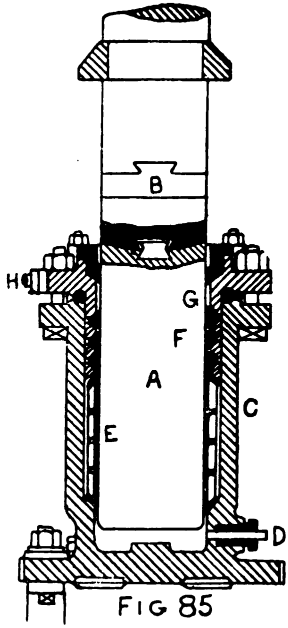


FIG 85

Fig. 84 show the top bearing, and fig. 85 the bottom bearing, in which A is a plunger supported by hydraulic pressure. These two carry the whole weight of the wheel. At the top of the former are the nuts A, A_1, with lock-screw and wedges D and E, by means of which the bell-shaped journal B is attached to the top of the shaft, and runs upon the bronze ring G. The load that is carried is 17·52 tons, consisting of the axial pressure of the water upon the wheel,

3·59 tons; axial pressure of the bevel wheels, ·4 ton; weight of shaft, &c., 13·53 tons. In order to reduce the friction, two annular grooves are turned on the face of C, and oil is supplied to them by a pump through the copper pipe F, the pressure being sufficient to ensure that B is carried by an oil film, and not merely by oiled bronze. The cup K catches the overflow, which is cleaned and used over again. In fig. 85, a cylindrical steel end G is attached to the bottom of the shaft by an Oldham coupling B, which is intended to allow for any eccentricity in the setting of the shaft. Water from an accumulator is forced into the cylinder C by the pipe D, and thus relieves the top bearing. The pressure exerted by the water is 18 atmospheres, giving a total upward pressure of 8·8 tons, so that the upper bearing carries 8·72 tons. If the packing in fig. 85 prevented the escape of water, the plunger A would get hot, as it has a high circumferential speed, and this would cause great danger to the packing. A leakage of about one quart of water per minute is therefore allowed, and the pressure is registered by a gauge, to which a pipe is led up from the cylinder C. Professor Radinger, of Vienna, was the designer of the lower bearing, the most interesting point about which was the packing, for no previous information existed as to how to maintain a fairly water-tight joint for a press whose plunger was in rapid rotation. The plunger A bears on its lower half a brass ring E, and above this some split conical rings F of an alloy. Obliquely placed leather rings lie under the gland at the top, while an indiarubber ring G is placed under the lower gland. The above account is from "Zeitschrift des Vereins Deutscher Ingenieure."

CHAPTER XVIII.

Theory of Radial-flow Impulse Turbines.

When the supply of water becomes very small, it is preferable to use a turbine in which there are very few guide passages, so that only partial admission takes place, and such turbines are generally of the radial outward-flow type, with horizontal axes, so as to drive direct or by belt, the water entering the wheel near the lowest point of the circumference, and leaving it *as near that point as possible*, so as to waste as little head as possible.

The construction for ϕ is the same as before, but only as a first approximation; so that

$$\phi = \pi - 2a,$$

and $$c_1 = v_1 = \tfrac{1}{2} v \sec a.$$

$$v = \cdot 94 \sqrt{2 g h_1}$$

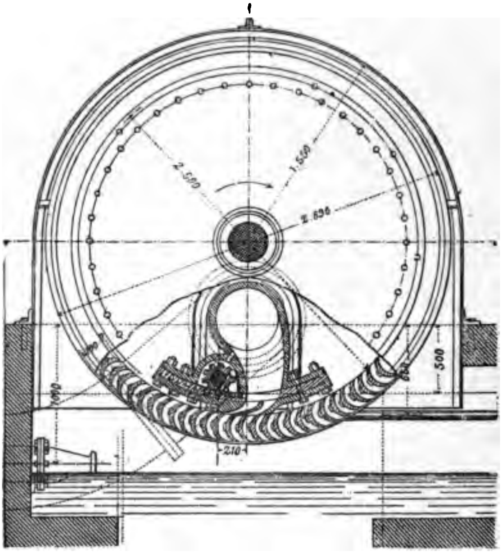


FIG. 86.

where h_1 = head above the centre of the guide passages;

$$a = \frac{9\,Q}{8\,v},$$

$$h_2 + h_1 = H.$$

The value of h_2 may be assumed as 1 ft., as before, but, of course, experience will be our best guide. It should be noted that h_2 does not necessarily equal $r_2 - r_1$, but should not differ much from it.

The outflow ought to be radial, but calculations from existing turbines show that generally it is not so; but

$$v^2 = c_1^2 + v_1^2 + 2\,c_1\,(w_1 - c_1)$$
$$= v_1^2 - c_1^2 + 2\,c_1\,w_1$$
$$u^2 = v_2^2 - c_2^2 + 2\,c_2\,w_2\,;$$

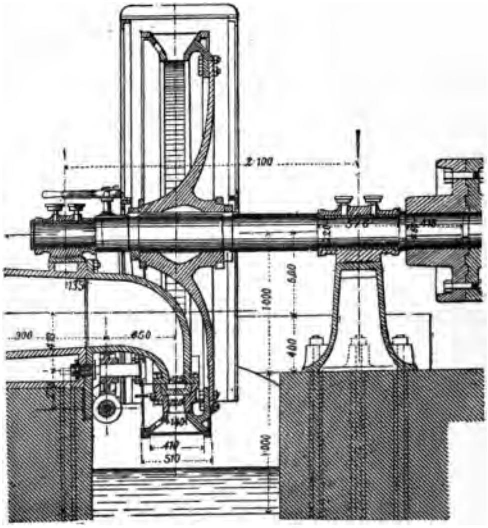


FIG. 87.

and considering the wheel alone, the energy available equals the work done plus energy wasted in hydraulic friction plus energy rejected.

$$\therefore \frac{v^2}{2g} + h_2 = \frac{c_1\,w_1 - c_2\,w_2}{g} + \frac{F_2\,v_2^2}{2g} + \frac{u^2}{2g}$$

where $\frac{F_2 v_2^2}{2g}$ is the energy wasted in hydraulic friction in the wheel.

$$\because v_1^2 - c_1^2 + 2c_1\,w_1 - v_2^2 + c_2^2 - 2\,c_2\,w_2 + 2g\,h_2$$
$$= F_2\,v_2^2 + 2\,c_1\,w_1 - 2\,c_2\,w_2.$$

$$\therefore (1 + F_2) v_2^2 = c_2^2 - c_1^2 + v_1^2 + 2 g h^2 \quad . \quad . \quad (32)$$

and F_2 may be taken as ·1.

$$c_2 = \frac{r_2}{r_1} c_1.$$

If the outflow were radial, (32) readily gives us

$$\cos \theta = c_2 \sqrt{\frac{1 + F_2}{c_2^2 + 2 g}}$$

assuming $c_1 = v_1 \; h_2 = 1$.

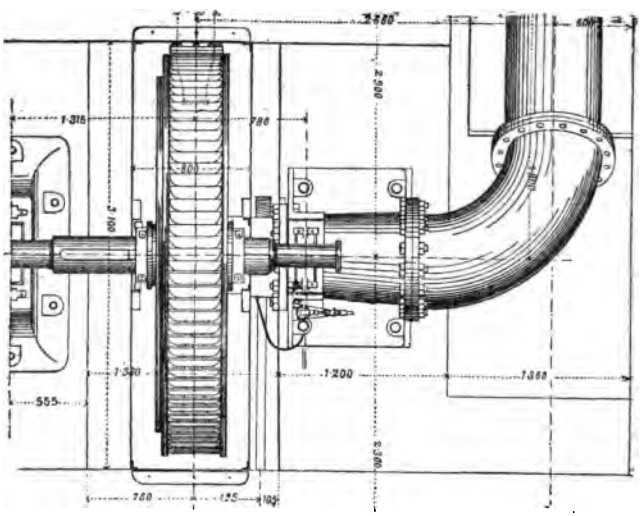


FIG. 88

But if $F_2 = ·1$, and c_2 is greater than 19·4, which is usually the case, this will make θ less than 14 deg., about its least practical value ; hence θ must be increased.

The following method of determining ϕ is suggested by the author partly from its agreeing with practice, and also because it does not violate theory. Impulse turbines are generally expected to have an efficiency of 75 per cent ;

hence their hydraulic efficiency may be taken as ·78. So if the outlow is radial—

$$\frac{c_1 w_1}{g} = \cdot 78 \text{ H}$$

$$c_1 = \frac{\cdot 78\, g \text{ H}}{v \cos \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and

$$\tan \phi = \frac{v \sin \alpha}{c_1 - v \cos \alpha},$$

so that ϕ can be calculated. This, however, will not always give a suitable value for θ from equation (32) when v_2 is put equal to $c_2 \sec \theta$, which is the condition for radial outflow.

We shall, however, use the above method for calculating ϕ, assuming θ, and calculating the loss of power caused by the term $c_2 w_2$ in the equation—

$$\text{Work done per pound} = \frac{c_1 w_1 - c_2 w_2}{g}.$$

To take a numerical example, let Q = 8·5 cubic feet per second, and H = 571, and let us suppose, to save calculation, that allowing for friction of supply pipe, &c., we obtain—

$$h_1 = 560$$

$$v = \cdot 94 \sqrt{2g \times 560} = 178$$

$$c_1 = \frac{\cdot 78\, g \text{ H}}{v \cos \alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Let

$$\alpha = 17 \text{ deg. } \cos \alpha = \cdot 596$$

$$c_1 = \frac{\cdot 78 \times 32 \cdot 2 \times 571}{178 \times \cdot 956}$$

$$= 84 \cdot 1$$

$$\tan \phi = \frac{v \sin \alpha}{c_1 - v \cos \alpha}$$

$$= \frac{178 \times \cdot 2923}{84 \cdot 1 - 170 \cdot 1}$$

$$= - \cdot 605,$$

being negative, ∴ ϕ is greater than 90 deg.

$$\phi = 148 \text{ deg. } 50 \text{ min.}$$

Let

$$r_1 = 3 \cdot 94$$

$$r_2 = 4 \cdot 56,$$

for the determination of which there are no mathematical formulæ.

$$c_2 = \frac{r_2}{r_1} c_1 = 97{\cdot}25$$

$$v_1 = \frac{v \sin \alpha}{\sin \phi} = \frac{178 \times {\cdot}2923}{{\cdot}5175} = 100{\cdot}5.$$

From (32),

$$v_2^2 = \frac{(97{\cdot}25)^2 - (84{\cdot}1)^2 + (100{\cdot}5)^2 + 64{\cdot}4}{1\,1}$$

$$v_2 = 106{\cdot}7.$$

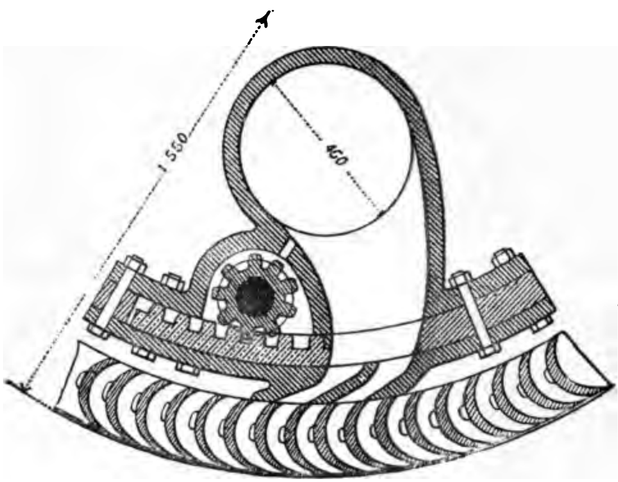


Fig. 89

For radial flow,

$$\cos \theta = \frac{c_2}{v_2}$$

$$= \frac{97{\cdot}25}{106{\cdot}7} = {\cdot}91$$

$$\theta = 24 \text{ deg. . } 31 \text{ min.}$$

This is somewhat larger than is usual, the maximum value being about 22 deg. The variation of θ for a small change in ϕ is very great, so that if ϕ be given values between 148

deg. 50 min., as above, and 146 deg., the latter value being obtained from the equation,

$$\phi = \pi - 2\,\alpha,$$

we shall find that $\cos\theta$ will increase up to unity and beyond it, so that for $\phi = 146$ deg. radial outflow would be impossible. Suppose—

$$\phi = 148 \text{ deg.}$$

$$c_1 = \frac{v \sin(\alpha + \phi)}{\sin \phi} = \frac{v \sin 15 \text{ deg.}}{\sin 32 \text{ deg.}} = 86{\cdot}9$$

$$c_2 = \frac{r_2}{r_1} c_1 = 100{\cdot}5$$

$$v_1 = \frac{v \sin \alpha}{\sin \phi} = \frac{v \sin 17 \text{ deg.}}{\sin 32 \text{ deg.}} = 98{\cdot}25,$$

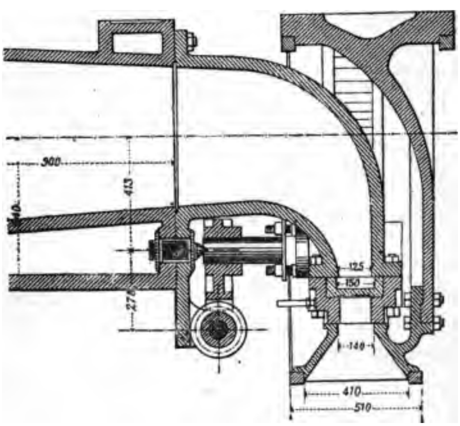


FIG. 90.

and from (32), using the above values, we obtain

$$v_2 = 105{\cdot}6$$

$$\cos\theta = \frac{c_2}{v_2} = {\cdot}9525$$

$$\theta = 17 \text{ deg. } 44 \text{ min.}$$

$$a = \frac{9}{8}\frac{Q}{v} = \frac{9}{8} \times \frac{8{\cdot}5}{178} = {\cdot}0536 \text{ square foot.}$$

This is so small that one guide vane is sufficient, so that there are two guide passages. Let the width of the guide

passages, measured parallel with the axis, be ·36 ft., and let the thickness of the vane be ·031 ft. If b be the breadth of the guide passages, and l the length of circumference taken up by them—

$$b\, l \sin \alpha = \cdot 0536 + \cdot 031 \times \cdot 36$$

$$l = \frac{\cdot 06475}{\cdot 36 \times \cdot 2923} = \cdot 616 \text{ ft.}$$

$$= 7{\cdot}4 \text{ in. nearly.}$$

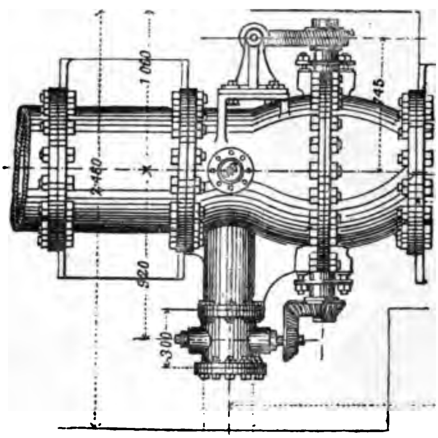


FIG. 91.

The fraction of the circumference through which the stream is passing is—

$$f = \frac{l}{2 \pi r_1} = \frac{\cdot 616}{\pi \times 7{\cdot}83} = \cdot 0249.$$

The breadth of wheel at inflow should be a trifle greater than b—in this case about ·42 ft.

Let $n = 100$, the number of vanes in the wheel, and let t_2, the vane thickness, be ·031 ft., as before, then we can use the formula—

$$f a_2 = K_2 \frac{Q}{v_2};$$

the quantity f being introduced because the admission is only partial; and K_2, which lies between 1·3 and 2·5, because the passages are not filled at discharge.

Then $$b_2 = \frac{a_2}{2\pi r_2 \sin\theta - n_1 t_2}.$$

It is best to choose a probable value for b_2, and calculate K_2 therefrom.

Let $b_2 = 1{\cdot}31$ ft.

Then $a_2 = 1{\cdot}31 \{2\pi \times 4{\cdot}56 \times {\cdot}3046 - 3{\cdot}1\}$

$$a_2 = 7{\cdot}335$$

$$K_2 = \frac{f v_2 a_2}{Q} = \frac{{\cdot}0249 \times 105{\cdot}6 \times 7{\cdot}335}{85}$$

$$= 2{\cdot}27.$$

Radial Flow Turbines at the Steel Works of Terni Italy.

At the steelworks of Terni there are eleven radial flow turbines, of horse powers between 1,000 and 20, the head available being 595 ft., and the quantities of water used per second being from about 20 cubic feet to ·42 cubic feet. They all run at very high speeds, from 200 to 850 revolutions per minute. They may be divided into two principal groups: (1) The small wheels of 20 to 50 horse power, which are mounted on a cast-iron frame, and can be removed and attached to the machines, to be set in motion as required; (2) the great motors, each placed separately on masonry and concrete foundations. We shall describe one of the second group, which works a mill for the production of railway rails; its horse power is 800. There are guides bolted on to a large pipe, which is fixed to a solid foundation, and from which a water pipe branches at the opposite end to the guide apparatus. This latter pipe is 1·96 ft. in diameter, and allows for a discharge of 15·89 cubic feet per second. On the figures the dimensions are given in millimetres, which must be multiplied by ·00328 to reduce to feet. The inner diameter of the wheel is 8·2 ft., and it makes 200 revolutions per minute, so that its circumferential speed is considerable. It is therefore constructed of very hard cast iron, whose breaking stress is over 8¼ tons per square inch, which gives a factor of safety of about 14. The rim is further strengthened by two steel rings, welded up and shrunk on, and it is bolted to the boss by a disc (figs. 86, 87, 90). The guide apparatus and means of regulation are shown to a larger scale (figs. 89, 90). There are two guide passages which may be partially or completely closed by the sluice above them. This sluice has teeth on its upper side, which

gear with a pinion upon whose shaft at the opposite end is a worm wheel (fig. 90) driven by a worm on the end of a shaft, which is visible in fig. 86, near the part of the wheel in section, and which is actuated by bevel wheels and a hand wheel, not shown. Two bearings carry the main shaft

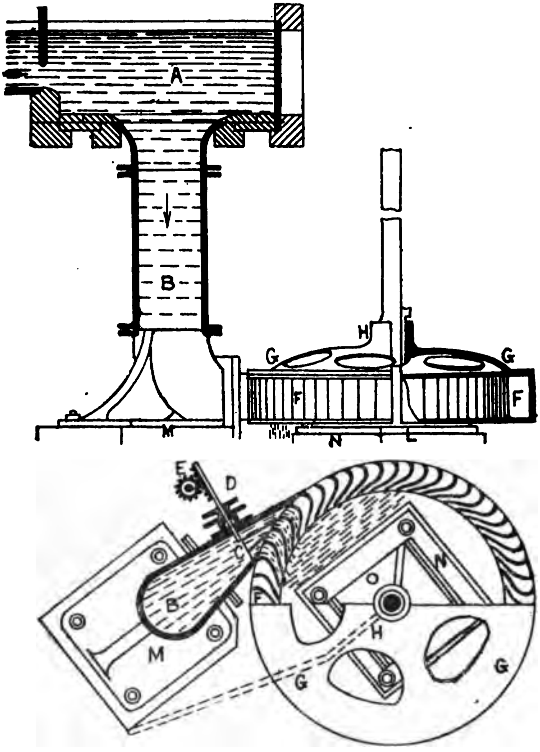


FIG. 92.

(fig. 87), in which are two grooves, hollowed to receive the steel rings which are shrunk on after fixing the wheel on the main shaft. It is necessary that the turbine should be stopped and started easily and quickly. It is not possible to stay the flow of water by means of the sluice, because of

the enormous pressure that would come on it. A stop valve (fig. 91) is used, from which a small pipe branches to the right, in which is a small valve, which, by means of bevel sectors, is opened when the stop valve is closed, so that the flow of water may continue, and no damage may be done to the pipes.

Zuppinger's tangent wheel (fig. 92) is an impulse turbine of the inward-flow type, which we introduce merely to show that this type has been tried and discarded in favour of the outward-flow class, when partial admission is necessary. This latter has the advantage that its axis may be horizontal, so that it may drive direct or by belt, and the only head lost is the clearance of the lower circumference of the wheel above the tail race. Without a suction tube, this would be impossible with any other type of turbine.

CHAPTER XIX.

Correction of the Vane Angles for Axial Turbines.

The correction at inflow may be made in exactly the same way as is shown in fig. 50, except that now ϕ is greater than a right angle, and v_1 increases from the outer to the inner

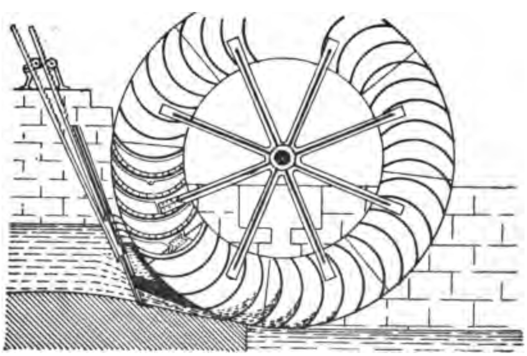


FIG. 93

radius; the correction at outflow is not the same as was described for reaction turbines. In the present case—

$$v_2 = \sqrt{\frac{v_1^2 + 2 g h_2}{1 \cdot 1}}$$

and $\therefore v_1$ and v_2 increase and decrease together. A simple construction, which the reader should now be able to make, will show that if θ is constant, the water will have a positive velocity of whirl at the outer circumference, and a negative or backward whirl at the inner radius, so that the mean velocity of whirl is zero. As in the case of the reaction turbine, it may be impossible to make the outflow radial at the outer circumference, because v_2 is not greater than c_2; but v_2 is always greater than c_2 at the inner radius, and correction might be made there. Generally, however, θ is constant.

THE "PONCELET" WATER-WHEEL.

Fig. 93 shows a type of wheel suitable to falls not greater than $5\frac{1}{2}$ ft. The stream flows under a sluice, and enters the wheel without shock, in consequence of the curvature of the vanes; it then rises up the vane, in consequence of its relative velocity v_1, and falls again, and flows out radially. The sluice is inclined to the horizon at an angle of about

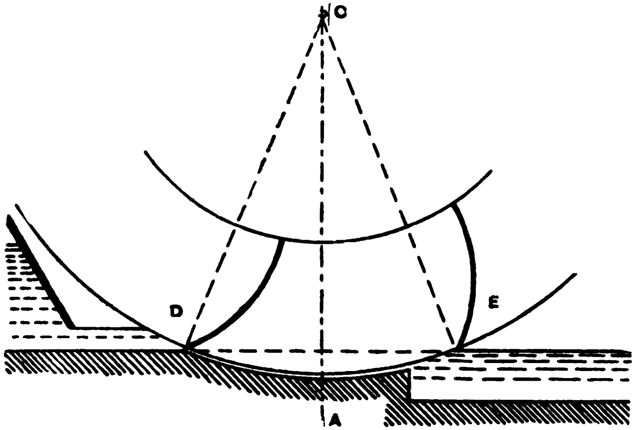


FIG. 93A.

50 deg. Some head must be lost if the wheel does not work in back-water, as fig. 93 shows the water leaving the wheel at some little height above the tail race. Theory therefore advises some such arrangement as that shown in fig. 93A,

where D E is a horizontal line, and the angle D C E is bisected by the line C A, so that the fall of the water, after it has attained its greatest height in the wheel, is the same as its rise. If we neglect friction, $v_2 = v_1$, $\theta = \pi - \phi$, and $c_1 = c_2$.

$$\therefore c_1 = v_2 \cos \theta$$

and
$$\frac{c_1}{\sin(\alpha + \phi)} = \frac{v_1}{\sin \alpha}$$

$$\therefore \sin(\alpha + \phi) = \sin \alpha \cos \theta = - \sin \alpha \cos \phi$$

$$\therefore 2 \sin \alpha \cos \phi = - \cos \alpha \sin \phi$$

$$2 \tan \alpha = - \tan \phi.$$

Generally, $\quad$ D C A = about 15°.

$$\therefore \alpha = 15°.$$

An efficiency of about 70 per cent has been obtained with this wheel, but this is too high for calculation, 66 per cent being more suitable.

CHAPTER XX.

THE PELTON OR TANGENTIAL WATER-WHEEL.

THIS wheel is shown in fig. 94.* It consists of a number of double buckets, fixed to the circumference of a wheel; water is projected from a nozzle, strikes the buckets in the centre, and is deflected equally to both sides. Let v be the velocity with which the water is projected, c the velocity of the

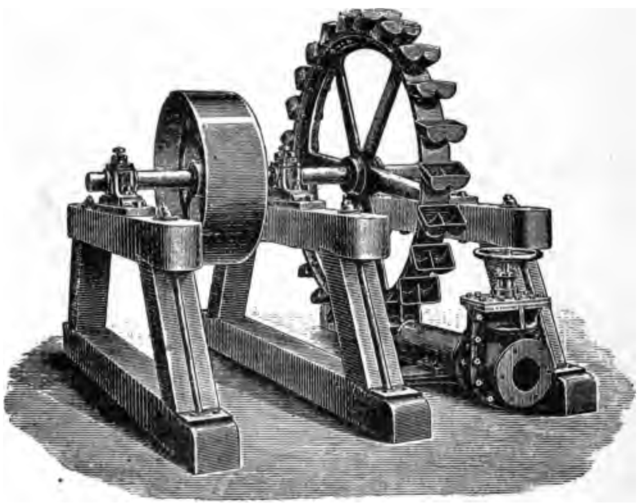


FIG. 94.

centre of the buckets, v_1 the relative velocity of inflow, and v_2 that at outflow. A section of a bucket is shown in fig. 95, and it will be seen that shock must take place at inflow, owing to the fact that theory demands that the bucket at the centre should be like a knife edge, which would be impracticable. Neglecting this, let us suppose that the

* From *Engineering*.

coefficient of resistance referred to v_2 is F, which gives us the following equations :—

$$\text{Work done by wheel} = \frac{c\,(w_1 - w_2)}{g};$$

where w_1, w_2 are the tangential absolute velocities of the water at inflow and outflow—

$$w_1 = v$$
$$w_2 = c - v_2$$
$$v_1 = v - c$$
$$\frac{v_2^2}{2g}(1 + \mathrm{F}) = \frac{v_1^2}{2g} = \frac{(v - c)^2}{2g}.$$

The total wasted energy is

$$\frac{\mathrm{F}\,v_2^2 + w_2^2}{2g},$$

which must be a minimum for maximum efficiency.

$$\therefore \frac{\mathrm{F}}{1 + \mathrm{F}}(v - c)^2 + (c - v_2)^2 \text{ is a minimum}$$
$$= \frac{\mathrm{F}}{1 + \mathrm{F}}(v^2 + c^2 - 2vc) + c^2 + \frac{(v - c)^2}{1 + \mathrm{F}} - \frac{2c\,(v - c)}{\sqrt{1 + \mathrm{F}}}.$$
$$\therefore \frac{\mathrm{F}}{1 + \mathrm{F}}(2c - 2v) + 2c - \frac{2\,(v - c)}{1 + \mathrm{F}} - \frac{2\,(v - c)}{\sqrt{1 + \mathrm{F}}} + \frac{2c}{\sqrt{1 + \mathrm{F}}} = 0.$$
$$\therefore c = \frac{v}{2} \text{ whatever the value of F.}$$

FIG. 95.

These wheels obtain a very high efficiency. Some tests made lately at the Mechanical Engineering Laboratory of the Ohio State University, Columbus, give the following results, the variation in speed causing the variation in efficiency.

Efficiency Tests.

	Head in pounds.	Head in feet.	Flow of water in pounds per minute	Revolutions per minute	Brake load.	Developed foot pounds of work.	Developed horse power.	Foot-pounds of work expended by water.	Efficiency.	Velocity of wheel. ft/sec
38 in. wheel.	75·55	165·3	3,330	309·8	80	495,680	13,662	550,400	90·04	47·75
	71·10	164·2	3,318	331·2	75	496,800	15,054	514,600	91·02	50·83
	71·75	165·7	3,335	273·6	90	492,480	14,924	552,500	89·16	41·66
	63·92	147·7	3,135	337·0	60	404,400	12,254	463,000	87·34	52·08
	67·25	155·3	3,222	336·0	65	436,800	13,236	500,350	87·30	52·02
	67·95	157·0	3,238	316·2	70	442,680	13,414	508,350	97·06	48·75
26 in. wheel.	70·15	160·0	2,075	482·8	30	289,680	8,778	331,980	87·23	49·80
	70·75	163·4	2,085	447·0	35	312,900	9,482	340,689	91·85	46·20
	70·35	162·5	2,078	377·4	40	301,920	9,149	337,675	89·40	37·95
	71·35	164·8	3,324	491·8	50	491,800	14,903	560,400	87·76	50·90
	71·95	166·2	3,340	445·2	55	489·720	14,840	555,108	88·20	46·25
	70·65	163·2	3,309	400·0	60	480,000	14,545	540,000	88·88	41·50

The types of wheel tested are shown in figs. 96, 97, being an improvement on that shown in fig. 94, because the buckets on each side of the continuous dividing edge catch the water alternately, thus securing a steadier motion. The impulses are divided more regularly on the wheel, as each bucket passes the point of the nozzle and catches its portion of the water. These buckets are cast solidly upon each side of the circular dividing ridge, and upon the face or rim of the wheel on each side of this central division. This circular ridge, being also angular and curved as it approaches the centre, gives to the interior of the buckets a symmetrical and effective curve. The further claim is made by the builders that this arrangement of buckets and form of construction secure great strength, firmness, and stability, and the buckets are not subject to the difficulty of becoming loose, as those styles in which bolts, nuts, and other appliances are used to fasten them upon the face or rim of the runner. In this type of wheel several jets are used. The mode of mounting is such as to permit of moving these jets,

FIG. 96.

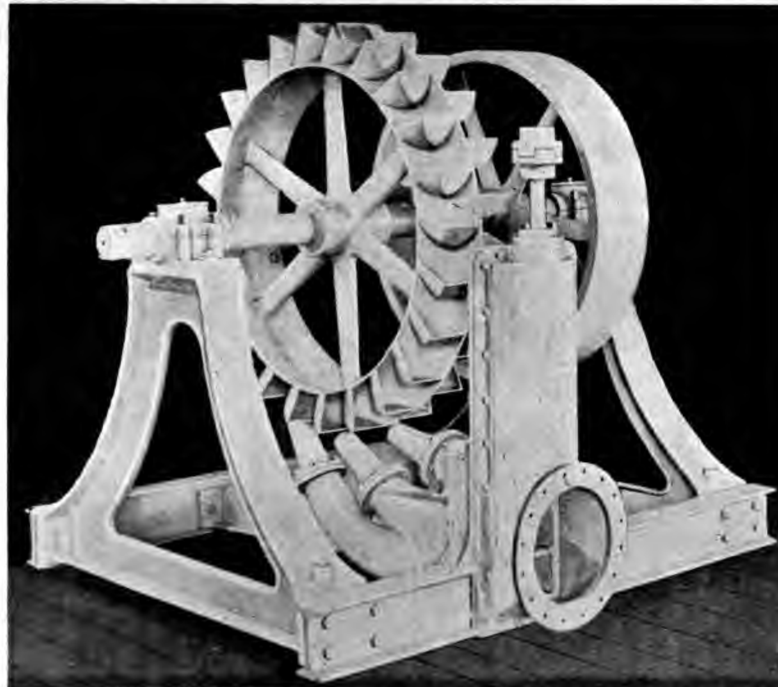


FIG. 97.

and thereby varying the inclination at which the water may be projected on the buckets. Either of the nozzles may be removed, and others of different size or bore put in their places, or any of them may be capped over, and one or several or all used, as may be desired.

CHAPTER XXI.

The Steam Turbine.

The design of the most successful form of steam turbine is that of Mr. C. A. Parsons. The action of a steam turbine is similar in many respects to an ordinary water re-action turbine, but there are two important difficulties to be overcome: the first is due to leakage, while the second is due to the enormous head under which the wheel works. The former is minimised by the methods we shall presently explain, and the latter, to a certain extent, by using a compound turbine, so that the head is divided between a number of turbines placed in series. In such motors it is impossible to obtain the minimum consumption of steam unless the clearances between the rotating blades and the enclosing conical disc, or cylindrical surfaces, and the fixed blades and rotating surfaces, is very small. This necessitates considerable delicacy in the adjustment and accuracy in the construction of the motors, and renders it difficult to keep them in such condition as to bearing surfaces that the best economy may be obtained in practical work through a long term of years.

In Mr. Parsons' inward-flow turbine, figs. 98, 99, and 100, a series of inward-flow wheels are arranged on one rotating shaft, and enclosed in a cylinder containing the guide blades, so that steam entering the first of the series passes successively through each before being discharged into the atmosphere or condenser by way of the port T. The turbine wheels consist of metallic discs, combined with bushes, which are slipped upon the shaft, and keyed or fixed to it in any convenient manner. Each disc carries one or more series of blades—in fig. 99 only one series is shown—so that the velocity of whirl is reduced to zero before leaving the wheel. The enclosing case carries ring projections and guide blades, and the ring projections are arranged so as to form a series of chambers, in which the turbine wheels

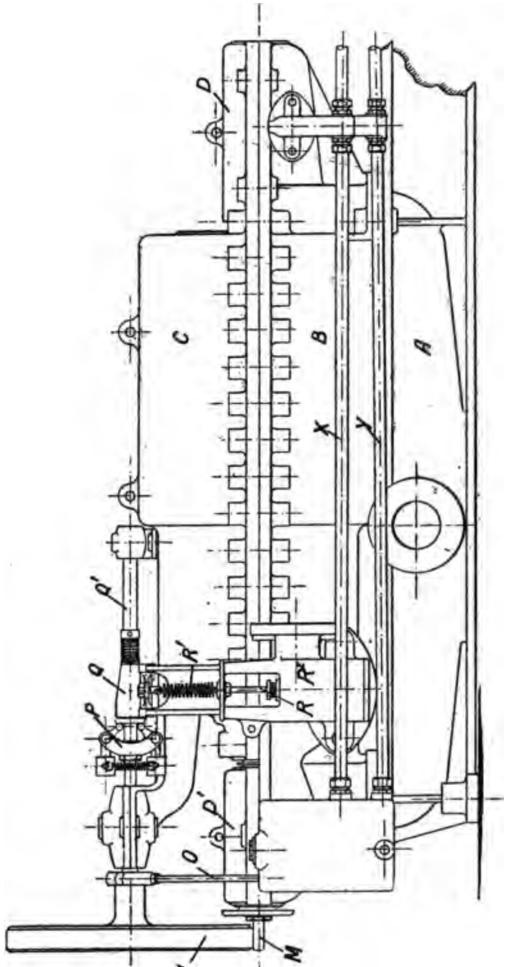


FIG. 98.

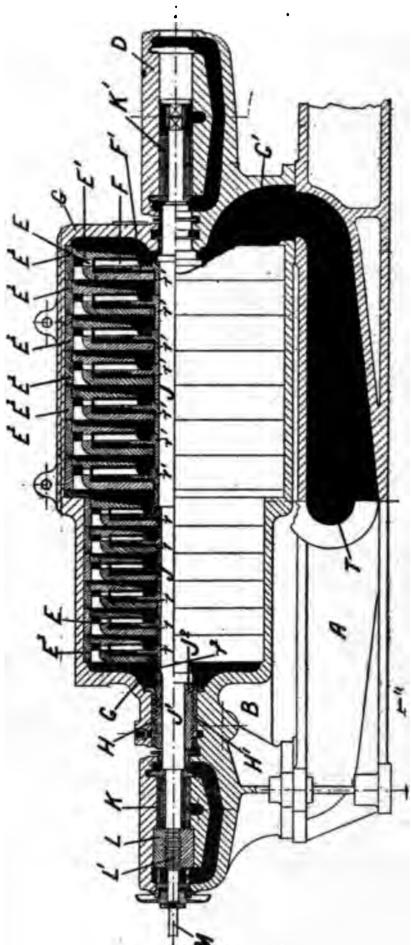


FIG. 99.

rotate. After the steam has reached the centre it is again directed outwards to the next series of guide blades, and thence to the next wheel. In order to prevent leakage from any wheel chamber to the next, whereby a portion of steam would be allowed to pass without flowing through the guide blades, the first partition ring E^2 of such chamber is carried close to the rotating bush carrying the wheel, or close to the

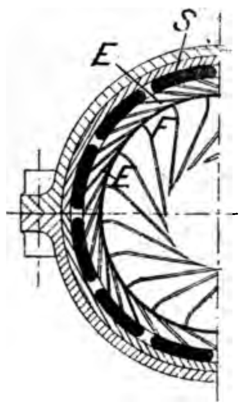


Fig. 100.

shaft itself, and grooves are turned in the portions of bush or shaft and ring opposing each other, alternately projecting into one another. In order to reduce leakage past the faces of the rotating vanes moving near the enclosing partition, a ring is frequently cast, or otherwise attached, to the lower edge of those vanes, so that the steam is baffled, and leakage reduced. At all surfaces where leakage is likely to occur tortuous surfaces are provided, which, of course, reduces leakage. The guide blades form parts of the partition rings.

In order to reduce vibration caused by a slight want of balance of the spindle, the following contrivance is made use of: the spindle ends run in a bush fixed into another bush with slight freedom of fit, and this into another, and so on, giving a number of concentric bushes having slight play the one within the other. The ends of these bushes are fitted into a case with some nicety of end fit, and the case is filled with oil, the outer bush being securely fixed. Any vibrating movement is now checked or damped by the

forcing out of oil between the bushes, so that although a slight movement is possible, yet it is resisted in whatever direction it may tend it to move.

In order to prevent leakage of steam past the spindle, a series of grooves or projecting collars are arranged upon the spindle, working easily in a similar series of grooves in an end bush, and by using a sufficient number leakage may be reduced to any desired extent. When there is an end thrust, which will, of course, be the case unless the steam is made to act upon two opposed turbines, it may be taken in a thrust block similar to that used in a marine engine, special care being taken that each collar takes its share of

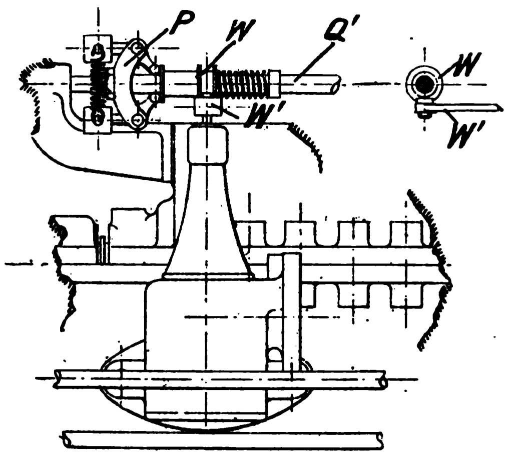


FIG. 101.

the pressure, so as to prevent heating at high velocities. The governing is accomplished in this case by a small shaft governor P, figs. 98 and 101, driven from the turbine spindle by friction gearing, and actuating a throttle valve, to diminish or increase the steam supply as required. The turbine spindle projects at M, fig. 98, and by frictional contact drives the friction wheel N on the second motion shaft Q^1, which shaft is thus rotated at a lower speed than the turbine shaft.

The centrifugal governor P controls the position of a conical sleeve Q on the shaft. When the speed increases,

the governor causes the conical sleeve to slide in one direction along the shaft, and when the speed falls the governor spring returns the sleeve again. The sleeve rotates with the shaft, and a cam surface cut on the cone moves a lever or other connection so that the steam valve is opened by its spindle R against the spring R^1. The conical or inclined cam surface is so arranged that in one position of the sleeve Q the steam valve is held open during the whole revolution of the shaft Q^1—that is, the steam is admitted continuously to the turbine by the steam valve. When the speed increases, however, the governor pulls the sleeve Q to such a position that the steam is admitted to

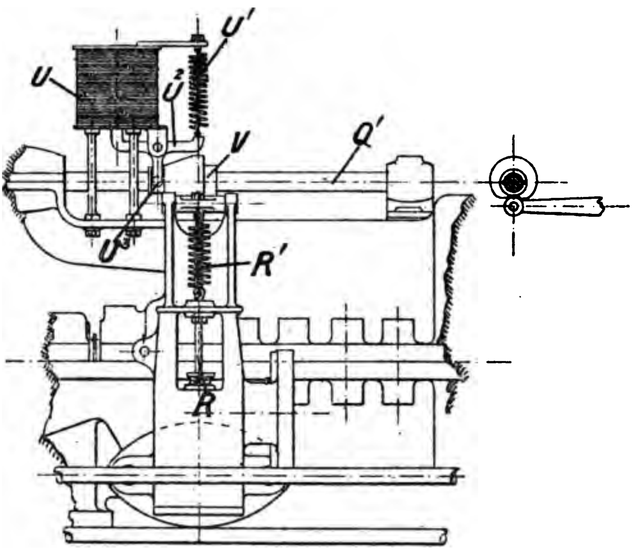


FIG. 102.

the turbine case for a portion of the revolution only, and the higher the speed the further the sleeve moves along the spindle, so that the steam is admitted to the turbine for a less and less fraction of a revolution of the shaft Q^1. This action of cutting off the steam resembles the variable cut-off of a reciprocating engine. The clearance space between

the first turbine and the stop valve is made as small as possible.

The use of a centrifugal governor may be obviated by the method shown in fig. 102. Here the solenoid arrangement U is actuated by the variation of the electric current, and increase or diminution of the speed of the turbine with its dynamo causes the lever U^2 to overcome the resistance of the spring U^1, or to be overcome by it. This moves the cam sleeve V on the second motion shaft Q^1 by the projection U^3, and so controls the steam valve as explained above. Fig. 103 shows another method where variable cut-off is dispensed with, and the variation of the electrical state of the dynamo caused by the solenoid U to act direct on the throttle valve R. In this case the solenoid core U^4 is fixed, and the coil is balanced on the end of the throttle valve

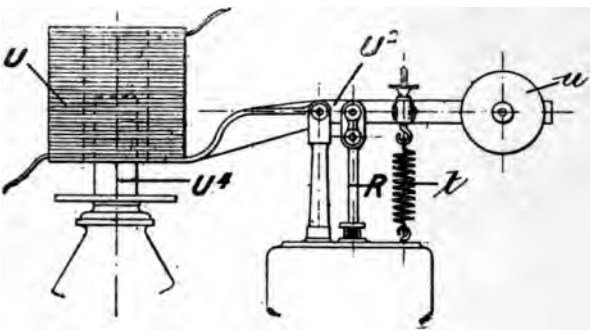


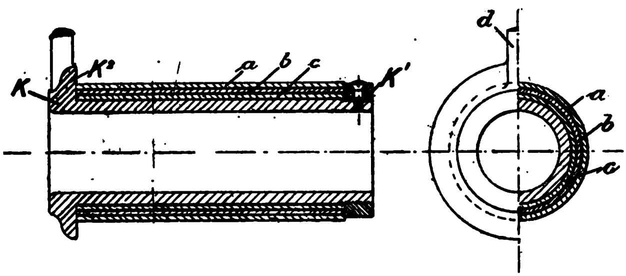
FIG. 103.

lever U^2. The throttle valve is opened by the spring t, and closed by the suck of the solenoid acting against the spring t and the weight u. The speed of the turbine may be varied by adjusting the tension of t, or by moving u. The application of a centrifugal governor rotating on the second motion shaft Q^1 to the control of the throttle valve is shown in fig. 101, where the governor P moves the sleeve W, and so varies the position of a throttle lever W^1, and admits or shuts off the steam.

The methods of damping vibration already referred to are shown in figs. 104 and 105. In the former the bush K, in which the turbine spindle runs, is surrounded by three concentric tubes a, b, c, preferably constructed of steel, and

turned so as to slip easily over the bush and each other with a slight freedom of fit. In the figures the looseness is somewhat exaggerated. The tubes *a*, *b*, *c* are held in position by a ring K^1, and move easily between the flange K^2 and the ring K^1. The outer tube *a* fits in the bored recess in D and D^1, figs. 98 and 99. In fig. 99 the two bearings of the turbine are shown in position at K and K^1. When oil gets access to the tubes *a*, *b*, *c*, which it is allowed to do by means of holes or grooves, a slight vibration set up by the spindle J, J^1 causes the tubes *a*, *b*, *c* to move or shake within each other. But the movement is damped by the hydraulic and capillary resistance of the oil between the rings, which must be squeezed out from between *a*, *b*, *c*, and at the ends K^2 K^1. The hydraulic resistance is considerable, and

Fig. 104.



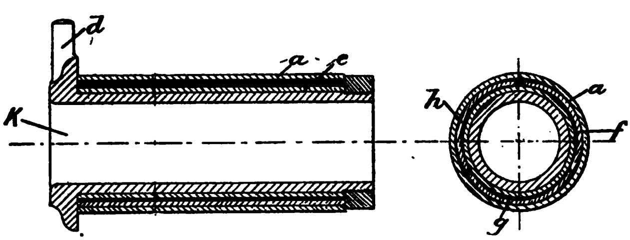


Fig. 105.

although some motion is permitted, yet it is resisted powerfully, and vibration is avoided. The arrangement shown in fig. 105 combines spring with hydraulic resistance, and it consists of two tubes *a*, *e*, surrounding the brush K, but having in the annular space between them spring segments

f, *g*, *h*, formed by cutting a tube of suitable diameter into three parts. These segments act as springs, freedom of lateral movement being allowed.

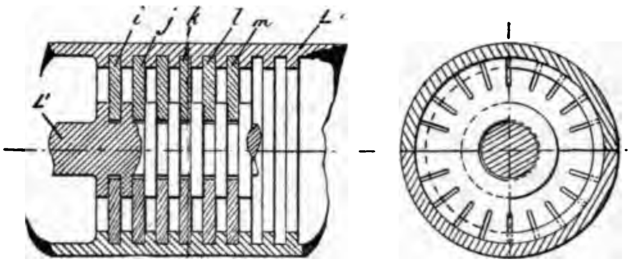


Fig. 106.

The unbalanced end thrust is taken up by means of the thrust block L, fig. 99, which is made in two halves, and slipped over the corresponding grooves and recesses in the spindle at L^1. This acts well in practice, but an alternative arrangement is shown in figs. 106 and 107, where the thrust

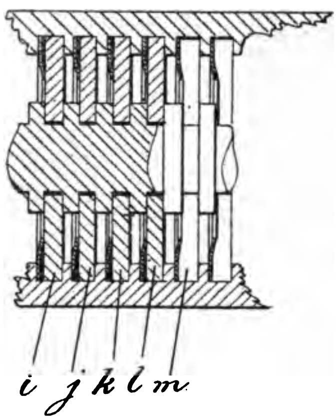


Fig. 107.

surfaces are pressed against the corresponding surfaces on the spindle by their elasticity, and so equally divide the total pressure amongst them. This elastic action is obtained

by constructing the thrust pieces i, j, k, l, m separately from the block L, of sufficient diameter and thickness to be elastic; slots cut out as shown in the right-hand view of 106 will give greater elasticity. The washers may also be held up to their work by springs, fig. 107.

The packing arrangements to avoid leakage of steam consist (fig. 108) of a bush H fastened on the spindle and contained in a block O, which is in halves, grooves n and projections p, p^1, p^2 being formed in both as shown, and by engaging prevent leakage.

The steam enters the turbine case by an inlet pipe, passing through the throttle or cut-off valve casing R^2, and thence to the space G, where the steam passes outside the first of the division rings E^2 by the fixed guide blades E to

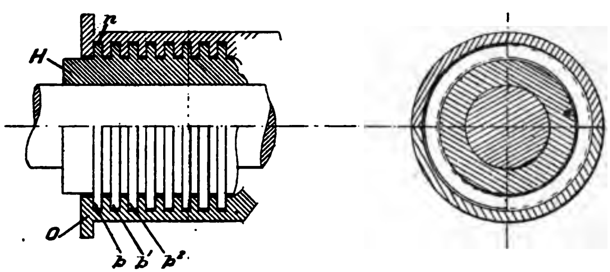


FIG. 108.

the wheel blades F on the first rotating wheel. It then passes after leaving the first wheel between the next partition rings E^1 and E^2 to the succeeding guide blades E, and thence to the wheel blades F, and in this manner passes through the whole turbine, the steam being expended and the pressure reduced to the required extent.

Views of a radial outward-flow turbine are shown in figs. 109 and 110. The spindle A rotates in elastic bearings, such as have already been described, and the bearing K carries the end of the armature spindle driven from A by the connecting piece a. Discs B, B^1, B^2, B^3, B^4 are keyed or otherwise fastened on the spindle, and rotate with it, and carry rings of blades on one face, and these alternate with corresponding rings of fixed blades attached to or forming part of C, C^1, C^2, C^3, C^4. The rotating balance piston E secured on the turbine spindle with the discs has turned ring facings and grooves which rotate in similar rings and

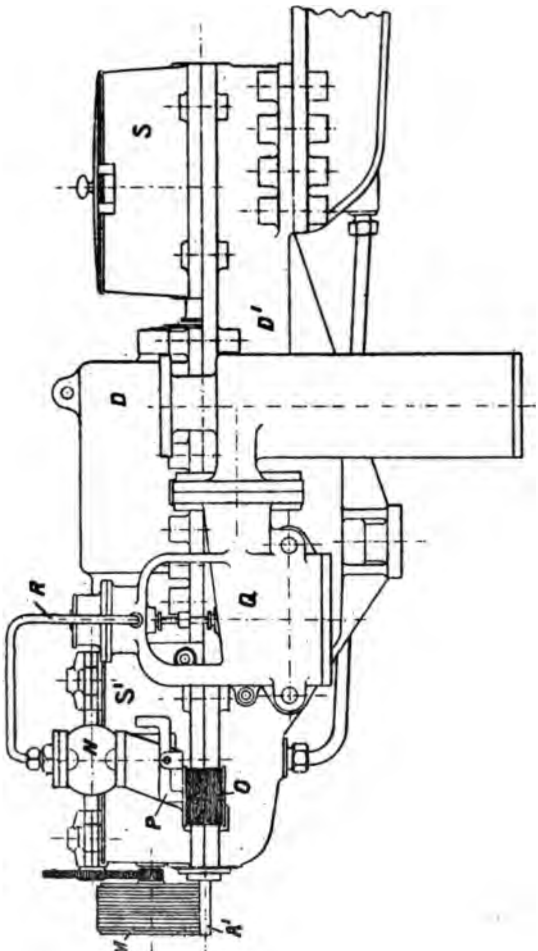


FIG. 109.

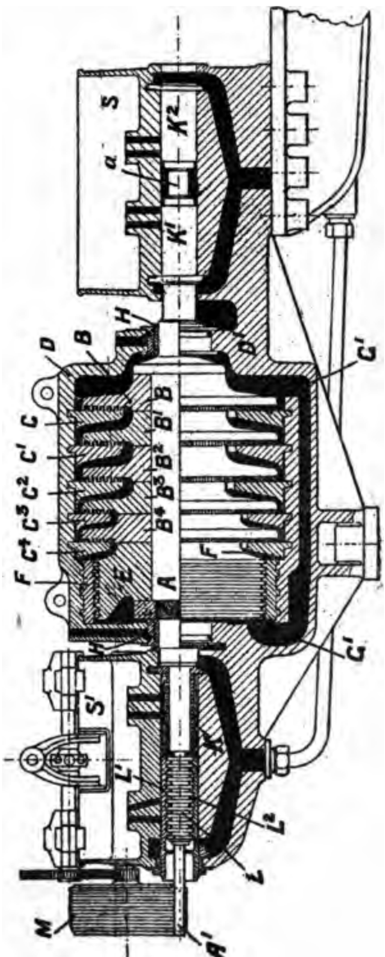
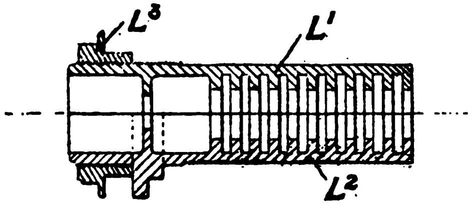


Fig. 110.

grooves in the ring piece F. The thrust block is in two parts L^1, L^2, holding the thrust collars L. High-pressure steam is admitted to the turbine into the space F, between the balance piston E and the rotating ring B^4. It first

FIG. 111.



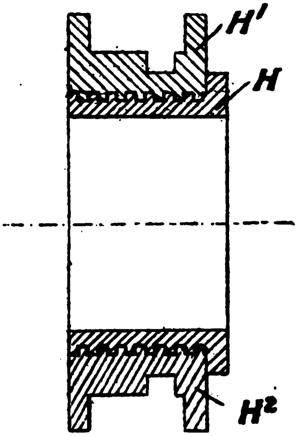


FIG. 112.

passes a row of fixed blades, and is directed outwards to rotating blades, whence it impinges upon a second fixed row and is directed to a second rotating row, and so on, until it discharges at the circumference of the first rotating disc B^4, and passes inwards to the first ring of the guide blades of the fixed ring C^2. The steam thus passes through the turbine, continually increasing in volume and falling in

pressure till it is discharged by the last row into the condenser or the atmosphere *via* the exhaust space G^1. This space is in free communication with the outer side of the rotating piston E, and so the external surfaces of the disc E and piston B are exposed to the same pressure. Where the spindle A enters the casing, packing is arranged to prevent leakage of steam with rings and facings, as shown at H, fig. 110, and on a larger scale in fig. 112. H fits tightly over the spindle A, and rotates with it within the sleeve H^1, made in halves and fixed to the case. The thrust bearing, or the bearing which maintains the longitudinal position of the spindle relatively to the turbine case, is of the collar-thrust type, fig. 111, and is arranged to admit of the longitudinal adjustment of the spindle A with the discs B, B^1, &c., and its balance piston E. The balance piston E is proportioned relatively to the disc, so that some preponderance of pressure tends to keep the turbine discs out of contact with the fixed discs, or to pull the spindle from L^1 to the other side of the casing. L^1, L^2 are two separate pieces, the latter being fixed to the casing, while the former can be adjusted by the nut L^3, the screw of which does not engage with any screw on the end of L^2. When L^3 is screwed up to the projection on L^2, the upper half of L^1 slides over L^2, and causes the collars to bear hard against the grooves in L^2. The grooved spindle L, and the grooved block L^1 L^2 therefore permit of longitudinal adjustment, and by easing the grooves in the block the clearance between the guide and wheel blade surfaces, and the discs to which they are fastened, may be reduced to a minimum. The grooves and rings of the piston E are by the same adjustment brought as close as required to the grooves and rings of the ring F. The packings H are also tightened up.

The governor is shown in fig. 113, and at N, in fig. 109. The pump cylinder A, shown in transverse section, carries the piston A^1, which is actuated by a crank or eccentric on a countershaft, driven from the turbine spindle. The piston on its outstroke takes in air into the cylinder through the inlet valve K, and compresses it and discharges it through the valve L^3 by the pipe I to the cylinder B, under the piston C. The piston C is thrust down by the spring G, acting between it and the cap G^1; and the piston by the piston rod C^1, passing through suitable packing glands, moves the balanced or double-beat valve D, which valve controls the admission of steam to the turbine by opening or closing the passage F leading to the turbine, and so allowing steam to enter from the live steam space E, or cutting it off.

In ordinary action each stroke of the pump piston, by forcing air under the piston C, causes it to rise against the spring G, and so open the valve D.

When the pump ceases forcing air, the leak-off aperture H, which is controlled by the screw H^1, permits the air to escape from the cylinder B, and so allows the spring G to thrust down the piston C, and close the steam admission

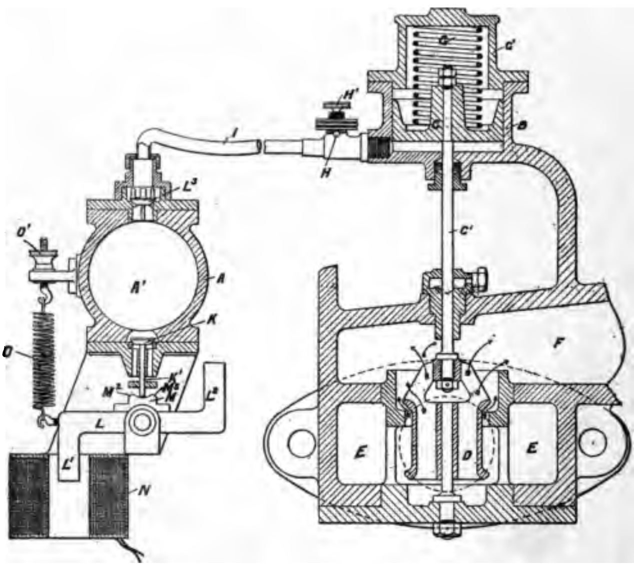


FIG. 113.

valve. The leak-off aperture H may be regulated to cause the valve D to close with the desired rapidity, so that it opens and closes at every stroke of the pump, or remains open till the governor determines that it shall close. One arrangement of electrical governing contrivance is shown. The solenoid N has pivoted above it the lever L, which carries at one end the immersed core L^1, forming part of the lever, and is balanced by the end L^2. The spring O, which is adjusted by the screw O^1, resists the action of the solenoid, and pulls the lever L to one extreme position when no current is passing. The spindle K^1 of the inlet or suction

air valve K projects below the cylinder, and when the lever L is in the position shown upon the drawing, the end of the spindle is just clear of the shaped or cam piece M, carried upon the lever L. The valve K then acts at every stroke of the pump, and so a charge of air in a compressed state is delivered to the cylinder B at every stroke. If, however, the speed of the turbine and dynamo should increase, the solenoid N will pull the core L^1 further in against the action of the spring O and the cam or valve K, so that the air taken into the pump on the outstroke is discharged on the return stroke without passing through the valve L^3, and therefore the piston remains down, and does not open the valve D. So long as the speed remains too high the valve D is kept shut. When the solenoid N ceases to pull the core L^1 to such an extent as to hold open the valve K, then the pump resumes its action, and by forcing air causes the valve D to open and close at every stroke, or remain more or less open, as determined by the adjustment of the leak-off aperture H. If by any accident the electric current should cease, then the solenoid N ceases to act on L^1, so that the spring O pulls the lever L to such a position that the projection M^2 raises the valve K. The pump is thus put out of action, and the steam valve D is closed. If the turbine is driving a dynamo giving constant potential, with varying quantity, the solenoid should be shunt-wound; but if the potential varies with the load, series coils should be added or subtracted. Should constant quantity but varying potential be required, the winding should be in series.

The parallel-flow turbine, shown in section in fig. 114, is now the most generally used, being somewhat more efficient than the radial-flow type. It differs from the latter form in that the moving blades are fitted round the outer circumference of what may be termed "a stepped barrel," comprising a series of barrels A, B, C, of different diameters, rigidly keyed to the turbine shaft. On the inner periphery of the turbine case are fitted corresponding fixed or stationary blades, between which the moving blades revolve. The barrels D, E, F are simply dummies, for the purpose of preventing end thrust, no effective work being done by them. In the figure it will be seen that there are three diameters of barrel, and that those of corresponding diameters are connected by passages in order to equalise the pressure. The diameter of the barrels, and the clearances between the blades, or steam passages, are varied to suit the increasing volume of the steam as it expands towards the low-pressure

end, thus keeping the velocity of the steam practically constant throughout. The steam is admitted through the double-beat valve H, and enters the turbine case all round the spindle at J, thence flowing in each direction towards the ends of the cylinder, the impact of the steam on the moving blades causing the barrel, and consequently the turbine shaft, to rotate.

The steam turbine may be advantageously applied to the driving of almost any class of machinery, either by direct coupling or by means of belts, ropes, or gearing.

On the 11th, 22nd, and 28th of January, Mr. W. D. Hunter, M.I.M.E., engineer to the Newcastle and District Electric Lighting Company Limited, carried out a series of tests on a 200 kilowatt continuous-current Parsons generator, for the purpose of determining the steam consumption under different conditions of service and various grades of output. The generator was designed to work with a fair all-round economy, whether exhausing into the atmosphere or into a condenser, although in general the latter method is expected to be employed. The difficulty of obtaining high economy imposed by such a set of conditions can only be partially met in ordinary engineering practice by adding to the engine a costly automatic expansion gear, which cannot always be relied upon when required to act within a widely-fluctuating range of load, or when called upon suddenly to work at full power, either high pressure or condensing. In the case of the Parsons turbo-generator the provision for expansion is constant, and when designed to exhaust into a condenser only the terminal pressure would be about $1\frac{1}{2}$ lb. to the square inch, with an initial boiler pressure of 140 lb., the steam being thereby expanded about a hundred times. The degree of economy obtained under these conditions is already well known; the last 150 kilowatt generator supplied to the Newcastle and District Electric Lighting Company required only 17·28 lb. of water per E.H.P. per hour at full load. When, however, the motor is required to give full-power working, either high pressure or condensing (with a moderate consumption of steam), the problem of how to meet conflicting requirements presents many difficulties, and the fact that these difficulties have been successfully met in the design of the generator tested, without any addition being made to the cost or parts of the machine, is further testimony to the adaptability of the steam turbine for every condition of service. The particular machine tested had one of the parallel-flow type of turbines coupled direct to a continuous-current dynamo, designed for a normal output of 200 kilo-

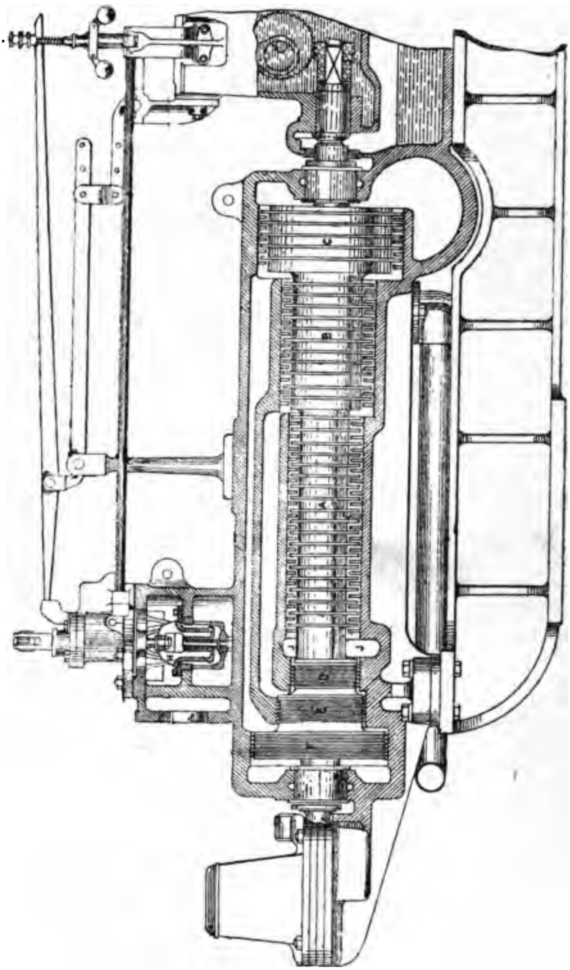


FIG. 114.

watts. Steam was admitted at one end of the cylinder, the admission being controlled by an exceedingly sensitive and effective electrical governor, which reduced or prolonged the period of admission without in any way altering the initial pressure at the steam chest of the motor; there was therefore a complete absence of throttling, with the attendant disadvantages which would be felt when working on a fluctuating load. The action of the governor left nothing to be desired, the rise in voltage being only momentary when the whole load of nearly 280 electrical horse power was thrown off. Between full load and no load the generator responded to whatever calls were made upon it without any hunting, this valuable property eminently fitting it for electric traction or haulage purposes.

During the trials measurements of electrical output, steam pressure, vacuum, &c., were taken each time the water from the measuring tanks was used up, the intervals at full load being about eight minutes. The measuring tanks from which the feed water was drawn were carefully calibrated, and the electrical output was taken by a Kelvin watt meter, the readings of which were checked by ammeters and volt meters. The figures obtained during the trials are shown in the following table. It will be noted that for the full-power trials the water used per electrical horse power hour when exhausting into the atmosphere was 32·22 lb., under similar conditions, but with the steam superheated 30 deg. Fah. the consumption was 30·97 lb. With saturated steam and exhausting into a condenser (vacuum 25 in.), the consumption fell to 19·51 lb. per E.H.P. hour. The generator ran throughout the trials smoothly and without a hitch, the automatic lubricating arrangements acting perfectly. The makers are Messrs. C. A. Parsons and Co., of Heaton Works, Newcastle-on-Tyne.

TESTS OF 200-UNIT TURBO-DYNAMO.

Kilo-watts.	Total water per hour. Lbs.	Water per kilowatts. Lbs. per hour	Water per E.H.P. Lbs. per hour.	
219·2	9,466	43·20	32·22	Non-condensing.
98·7	5,848	59·23	44·18	
54·5	4,330	79·50	59·30	
0	2,092	...	...	
203·0	8,429	41·52	30·97	Non-condensing, and superheating 30 deg. Fah.
106·1	5,287	49·83	37·17	
0	1,402	...	...	

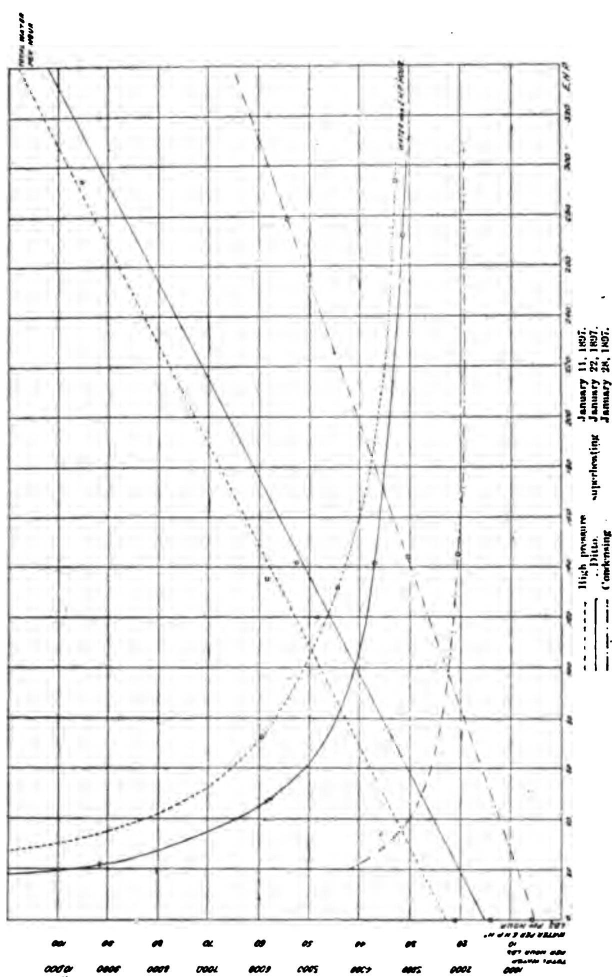

High pressure January 11, 1897.
Ditto superheating January 22, 1897.
Condensing January 28, 1897.

FIG. 115.

Kilo-watts.	Total water per hour. Lbs.	Water per kilowatts. Lbs. per hour.	Water per E.H.P. Lbs. per hour.	
208·0	5,443	26·16	19·51	Condensing, but no superheating. Vacuum at full load 25 in.
108·4	3,037	28·02	20·90	
0	531	...	...	

The above is taken from a paper by Mr. Parsons on the "Steam Turbine," published in the Minutes of the Institution of Mining Engineers.

CHAPTER XXII.

Comparisons between Theory and Experiment.

In the following examples we have chosen only such experiments as give inflow without or with very little shock. The reason for doing so has been to avoid as much as possible the more complicated calculations that would be otherwise necessary. It frequently happens that in practice inflow without shock and radial outflow are not obtained at the same speed, and we must therefore take into account the velocity of whirl w_2 at discharge.

Before commencing the comparison, we must explain the American method of measuring the areas for outflow from guide and wheel passages. Suppose a pair of inside calipers, placed with the point of one leg at the end of the vane, and the other point so that it just touches the next vane; then the area of that passage is its breadth, multiplied by the measurement given by the calipers; so that the total area is obtained by multiplying this area by the number of guide or wheel passages. This method of measuring areas gives a better agreement between theory and practice for radial-flow turbines, but for the axial-flow type the two methods lead to almost the same result.

The three examples we shall take are from Prof. Thurston's paper on "The Systematic Testing of Turbine Water-wheels in the United States," published in the Transactions of the American Society of Mechanical Engineers, vol. viii. They were made at Holyoke, Mass., where the most complete arrangements are made for turbine testing, and the most accurate experiments have been carried out.

It would take too much space to give these tests in full, and it will be sufficient to say that the wheels were tested at varying speeds, and with different gate openings, and that the examples chosen are very close to those giving maximum efficiency.

The first experiment is that made with a Collins axial turbine, in which $\alpha = 17\frac{3}{4}^\circ$, $\phi = 77\frac{3}{4}^\circ$, $\theta = 19^\circ$, $r = 2·085$, $n = 24$, $n_1 = 30$; guide area $a = 2·912$ square feet; bucket area $a_2 = 2·822$; Q = 64·88; H = 16·56; revolutions per minute = 105·5, whence we obtain—

$$v = \frac{64·88}{2·912} = 22·3$$

$$v_2 = \frac{64·88}{2·822} = 23$$

$$c = 2\pi r \frac{N}{60}$$

where N = revolutions per minute—

$$c = 2\pi \times 2·085 \times \frac{105·5}{60} = 23.$$

If ϕ had been such that inflow took place without shock—

$$\tan \phi = \frac{v \sin \alpha}{c - v \cos \alpha}$$

$$= \frac{22·3 \times ·305}{23 - 22·3 \times ·9523} = 3·89.$$

$\phi = 75^\circ\ 7'$, which differs very little from $77^\circ\ 45'$, so that shock at inflow may be neglected.

$$w_1 = v \cos \alpha = 22·3 \times ·9523 = 21·25.$$

$$w_2 = c - v_2 \cos \theta = 23 - 23 \times ·9455 = 1·25.$$

Neglecting friction of bearings—

$$\text{H.P.} = \frac{62·5\ Q}{550} \times \frac{c\,(w_1 - w_2)}{g}$$

$$= \frac{62·5 \times 64·88}{550} \times \frac{23 \times 20}{32·2} = 105·1.$$

The actual H.P. was a trifle less, 101·25; this allows for bearing friction and for the usual disagreement between theory and practice. The maximum efficiency was obtained

at a less speed, viz., 96 revolutions, with 64·99 cubic feet per second, and 16·55 ft. of head. This gives rise to shock at inflow and a backward discharge, so that w_2 is negative, while in the above example w_2 is positive, and is the cause of a loss of over $6\frac{1}{2}$ H.P., which would have given maximum efficiency if added to the above.

The next experiment is that of an outward-flow Fourneyron turbine with a Boyden diffusor, a ring whose section is shown at A B, fig. 116, which surrounds the wheel and by its increasing section decreases the discharge velocity, and therefore diminishes the loss of head. It may be at first difficult to see how this can affect the wheel. The explanation is that if v_3 be the velocity at B and u that at A, and

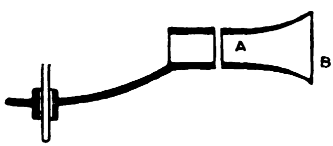


FIG. 116.

p_3 and p_2 be the corresponding pressures, then, if we neglect friction,

$$\frac{p_2}{62{\cdot}5} + \frac{u^2}{2g} = \frac{p_3}{62{\cdot}5} + \frac{v_3^{\,2}}{2g}$$

$$\frac{p_3 - p_2}{62{\cdot}5} = \frac{u^2 - v_3^{\,2}}{2g}$$

But v_3 is less than u, and so p_2 is less than p_3, the pressure of the atmosphere + the pressure due to the immersion of the points A and B. Thus the head between the upper surface and the point A is increased by the amount $\frac{u^2 - v_3^{\,2}}{2g}$ and therefore more power is available for the wheel. The test of this turbine was carried out at Holyoke, on April 26, 1882. The following dimensions are given: $\alpha = 24°$, $\phi = 90°$, $\theta = 26°$, $n = 34$, $n_1 = 54$, the guides and buckets being of brass, $a = 6{\cdot}814$ square feet, $a_2 = 5{\cdot}66$ square feet, $2\,r_1 = 73{\cdot}6$ in., $2\,r_2 = 90$ in. The maximum efficiency was obtained at $63\frac{1}{2}$ revolutions per minute, corresponding to a value of $c = 20{\cdot}4$ and a deflection of $8°\ 31'$ of the relative velocity of inflow; that is, for inflow without shock ϕ should be $98°\ 31'$, and it is actually $90°$. The

maximum efficiency was 80·17, and at $66\frac{1}{2}$ and 71·12 revolutions it had only fallen to 78·79 and 78·66 respectively, while the change of direction of the relative velocity at inflow was reduced to 4° 39′ and 2° 52′ respectively. We are therefore justified in comparing these two experiments with theory, neglecting any shock. Taking, firstly, that at 71·12 revolutions,

$$Q = 149{\cdot}87 \text{ cubic feet per second}$$

$$H = 16{\cdot}64$$

$$v = \frac{Q}{a \times {\cdot}9} \text{ taking a coefficient of contraction of } {\cdot}9 \text{ for radial turbines}$$

$$v = \frac{149{\cdot}87}{6{\cdot}814 \times {\cdot}9} = 24{\cdot}5$$

$$w_1 = v \cos \alpha = 24{\cdot}5 \times {\cdot}9135 = 22{\cdot}3$$

$$c_1 = 2\pi r_1 \frac{N}{60} = \pi \times \frac{73{\cdot}6}{12} \times \frac{71{\cdot}12}{60} = 22{\cdot}8$$

$$\tan\phi = \frac{v \sin \alpha}{c_1 - w_1} \text{ if inflow takes place without shock}$$

$$= \frac{24{\cdot}5 \times {\cdot}4067}{{\cdot}5} = 19{\cdot}9$$

$$\phi = 87^\circ\, 8', \text{ which differs from its actual value of } 90^\circ \text{ by } 2^\circ\, 51'$$

$$v_2 = \frac{149{\cdot}87}{5{\cdot}66 \times {\cdot}9} = 29{\cdot}5$$

$$c_2 = 22{\cdot}75 \times \frac{90}{73{\cdot}6} = c_1 \times \frac{r_2}{r_1} = 27{\cdot}9 \text{ nearly}$$

$$w_2 = c_1 - v_2 \cos\theta = 27{\cdot}9 - 29{\cdot}5 \times {\cdot}8988 = 27{\cdot}9 - 26{\cdot}5 = 1{\cdot}4.$$

Neglecting shaft friction,

$$\text{H.P.} = \frac{62{\cdot}5}{550} Q \frac{c_1 w_1 - c_2 w_2}{g}$$

$$= \frac{62{\cdot}5 \times 149{\cdot}87 \times [22{\cdot}8 \times 22{\cdot}3 - 1{\cdot}4 \times 27{\cdot}9]}{550 \times 32{\cdot}2} = 248.$$

If we assume that 3 per cent of the available power is lost by shaft friction, then, since the available horse power is $\frac{62{\cdot}5 \times 149{\cdot}87 \times 16{\cdot}64}{550} = 283$, the amount lost by the above cause is 8·49 horse power.

Then $$248 - 8{\cdot}49 = 239{\cdot}51$$

The actual horse power was 222·5, and the difference between actual and calculated is 17·01, which is 6 per cent of the available power, and 7·66 per cent of the actual power, and is not a very serious discrepancy.

In the next experiment, at 66·5 revolutions,

$H = 16{\cdot}62,\ Q = 148{\cdot}32$

$c_1 = 21{\cdot}3,\ v = 24{\cdot}2,\ w_1 = 22{\cdot}1$

$\tan\phi$ calculated $= -12{\cdot}3.$

$\phi = 94^\circ\ 39'$, so that the actual divergence is $4^\circ\ 39'$.

$c_2 = 26{\cdot}1,\ v_2 = 29{\cdot}25,\ w_2 = -{\cdot}15$, and H.P. = 248 nearly.

The actual horse power was 220·28, and the difference will be much the same as before, the greater shock at entry helping to account for the discrepancy between theory and practice.

The third experiment given in this paper is that of the Hercules mixed-flow turbine, of which, unfortunately, insufficient data are given from which to calculate the manner in which the water leaves the wheel. Not having sufficient data as to this class of wheel, we have avoided the theory of the subject here, the difficulty being to decide as

FIG. 117.

to the manner in which the outflow takes places from the peculiar and differently shaped buckets. Theory, unless supported by experiment, is best left until the necessary trials can be made. Fig. 117 is a perspective view of the wheel, the outflow from which is more inward than downward. There are divisions of guide and wheel passages by planes perpendicular to the axis of the wheel, so that the part-gate efficiency may not be reduced. The sluice is cylindrical, and its motion is parallel to the axis. It is not *surprising to* find that the maximum efficiency is at part

gate. The first test was made on August 13th and 14th, 1883, and gave such exceedingly good results that two others were made, so that we have every reason to believe in its accuracy. The dimensions given by Professor Thurston are as follow: $a = 14° 45'$, $\phi = 98°$, $n = 24$, $n_1 = 17$, $a = 4·752$, $a_2 = 7·925$ square feet, $r_1 = 1·5$ ft., and in the first of the three tests the maximum efficiency was 86·94, and was obtained at ·8 gate. In finding the velocity v, a must be multiplied by ·72, to allow for contraction and ·8 gate.

The speed selected is 136·5 revolutions per minute, at which the efficiency was 86·47, with Q = 78·34 and H = 17·35, while the maximum efficiency was obtained with Q = 78·44 and H = 17·35 at 131·5 revolutions per minute.

$$v = \frac{78·34}{4·752 \times ·72} = 22·9$$

$$c_1 = 21·45$$

$$w_1 = 22·9 \times ·967 = 22·15.$$

If we calculate ϕ from the formula,

$$\tan \phi = \frac{v \sin}{c_1 - w_1}$$

we get $\phi = 96° 53'$, compared with 98° actually; hence there is very little shock. If we neglect the quantity $c_2 w_2$,

$$\text{H.P.} = \frac{62·5 \text{ Q } c_1 w_1}{550 \times 32·2} = \frac{62·5 \times 78·34}{550} \times \frac{21·45 \times 22·15}{32·2}$$

$$= 131$$

The actual horse power was 133·17, but as we do not know how the outflow takes place, it is no use speculating as to the causes of difference between the actual and calculated horse powers. These are all the experiments described in detail in Professor Thurston's paper, and we wish the reader to understand clearly that we have not selected these in preference to others because they give closer agreement between theory and practice, but because they were made with the greatest accuracy, and the wheels are of modern construction. Professor Thurston's paper, mentioned above, need only be read to appreciate that accuracy. Again, in selecting tests at particular gates and revolutions, we have been solely guided by the efficiencies obtained and the absence of shock at entry.

These and other experiments show the best agreement between theory and practice when the areas are measured as explained above. This method of measurement has

another advantage, viz., that there is no need to calculate the areas, for they can be measured from a drawing. Thus, suppose we have calculated a and a_2 in the manner described above, and have settled our value of r_1 and r_2, we may make

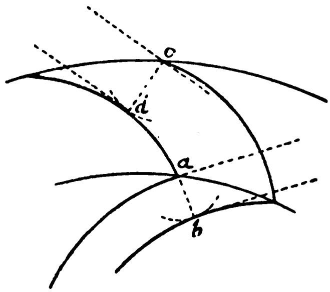


FIG. 118.

a drawing showing a section through wheel and guide passages, as in fig. 118. Then the breadth of the guide passages is b and $b \times$ measured distance $a\,b = \frac{a}{n}$.

Similarly $b_2 \times$ measured distance $c\,d = \frac{a_2}{n}$.

In the case of passages with involute vanes, measurement will give the same value as calculation by formulæ (24) and (25), page 59, neglecting the obstruction caused by the wheel vanes at inflow.

CHAPTER XXIII.

THE CENTRIFUGAL PUMP.

THIS type of pump is used for low lifts, but it has been also known to work economically with lifts as high as 40 ft. Reciprocating pumps are not economical for low lifts, and their first cost is greater than that of a centrifugal pump. The latter, however, require to be filled before they can pump, either by placing them below the lower surface of water, or by creating a vacuum by means of a steam ejector, which exhausts the air from the pump, into which the water then rises.

Figs. 119 and 120 show two sectional elevations of a centrifugal pump with horizontal shaft. When pumping has commenced, the water enters at A, and flows up the two side passages to the eye B of the pump disc or fan, the water entering at both sides. As the disc is rotating in the opposite direction to the hands of a watch, and as it is necessary that the water should enter it without shock, as in the case of a turbine, the vanes must be curved back at the inner diameter of the fan, because the water cannot have

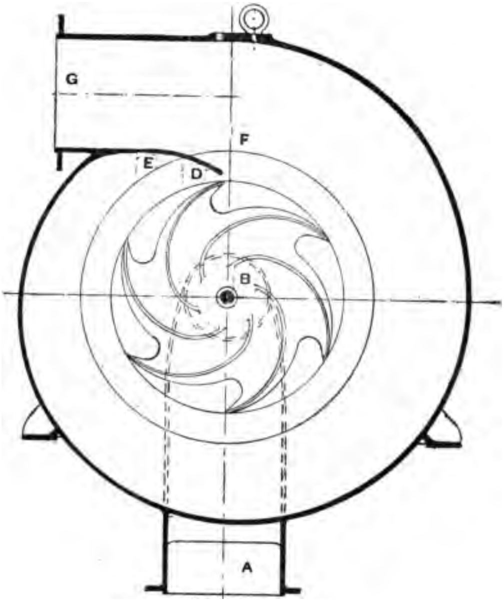


FIG. 119.

much, if any, velocity of whirl on entry. The vanes are still more inclined to a radius at the outer circumference, the angle being sometimes about 75 deg. There is, however, considerable difference of opinion as to the correct value of this angle, and it is possible for a smaller angle to be used under certain circumstances, without much sacrifice of efficiency. The discussion of this point we shall, however,

leave to the theory of the centrifugal pump, merely stating that most makers prefer a large angle, as in fig. 119. The water passes through the disc, its tangential velocity being

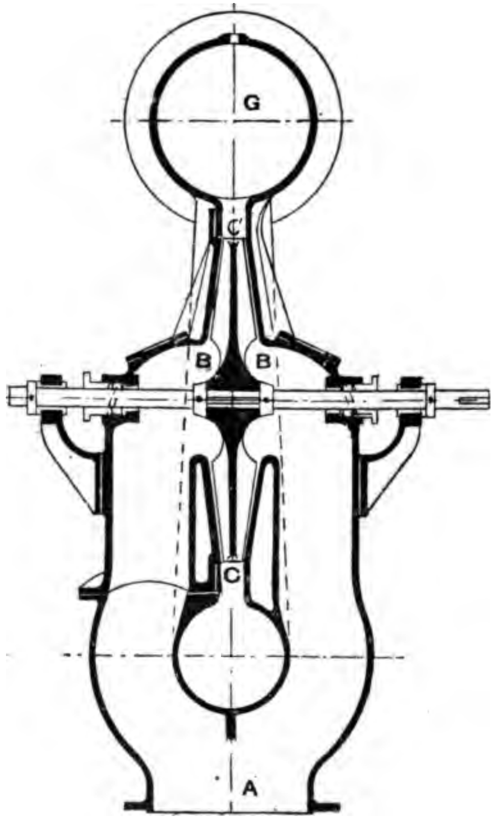


FIG. 120.

considerably increased at discharge. The work done by the pump disc depends mainly on two things—the velocity of its circumference and the tangential velocity of the water at discharge, and accordingly as the casing is well or badly

designed there are smaller or greater losses from shock after leaving the disc.

At C C (fig. 120) is a small whirlpool chamber or diffusor, in which part of the kinetic or velocity energy of the water is converted into pressure energy, and from this the water passes into the volute or spiral chamber, whose section increases uniformly from E round the circle; and as the quantity of water flowing into it from the disc increases in the same way, it is clear that the velocity is constant in this chamber. Discharge finally takes place at G. Sometimes conical suction and discharge pipes are used, the latter especially being of advantage, as we shall subsequently show, while the former allows of a gradual increase of velocity, which is always an advantage. In the earlier types of centrifugal pumps no casing or volute was used, or only a casing, which was of uniform section; and as there is a certain velocity round the outer circumference of the disc that causes the least waste of energy by shock, it is evident that this was one of the causes of their inefficiency. Most pumps are now made without a large diffusor, as it increases the diameter and weight, without increasing the efficiency to an extent sufficient to counterbalance these disadvantages. The fan can be got at by removing a cover shown at the left of fig. 120, which contains part of one of the suction passages. These pumps are generally driven directly by a small vertical or horizontal engine. Fig. 121 shows two pumps with engine for supplying the circulating water to a surface condenser. They are frequently used for this purpose on board ship, as they have the following advantages over reciprocating pumps:—

1. If they are in pairs, the one can be cleaned or repaired while the other is working without stopping the engines, which cannot be done with reciprocating pumps as usually fitted to and worked by the main engines.

2. A supply of water can be pumped through the condenser tubes while "blowing through" before the main engines start, so that the condenser is not overheated, and a good vacuum is at once obtained.

3. The head against which the pump works is small, and therefore the efficiency of the centrifugal is greater than that of a reciprocating pump.

4. Its action is continuous, and no valves or air vessels are needed in consequence.

5. Its discharge may be varied by increasing or reducing its speed.

Fig. 122 shows a side view of a pump with cover removed.

CHAPTER XXIV.

Theory of the Centrifugal Pump.

The above description will enable the reader to understand the following theory. In the first place, however, we must assume, for simplicity, that the axis of rotation is vertical. This assumption is required because, otherwise, particles at equal distances from the shaft would have different velocities and be under different pressures, which would complicate the theory, although the effect in practice is unimportant.

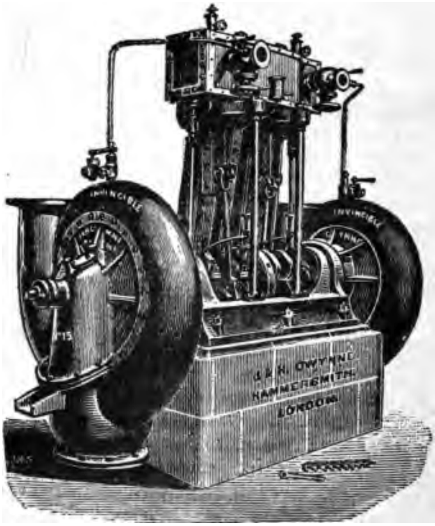


Fig. 121.

In fig. 123, B A is the curve of a vane, and $B c_2 u v_2$, $A c_1 v v_1$ are the parallelograms of velocity at the inner and outer radii of a particle of water entering the disc without sudden change of direction, and leaving it at A with a velocity $A v = v$ in a direction inclined to an angle α to the tangent $A c_1$. $A c_1 = c_1$, and $B c_2 = c_2$, the velocities of the disc at radii O A, O B, which we shall call r_1 and r_2. $B v_2 = v_2$, and $A v_1 = v_1$ are the relative velocities at entry and discharge.

Aw_1, Bw_2 are the velocities of whirl; w_1 and w_2, and Bu_2, Au_1 are the radial components u_2, u_1 of the actual velocities u, v at entry and discharge. The angles θ, ϕ should also be noticed, and the fact that if w_2 is zero β is a right angle.

The work done per pound during the passage through the disc is

$$\frac{1}{g}(w_1 c_1 - w_2 c_2),$$

so that if the direction of entry is radial, this becomes $\frac{w_1 c_1}{g}$ and even if it is not radial, and a vortex has been set up in

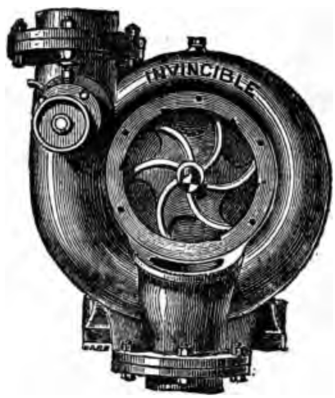


FIG. 122.

the eye of the pump, the total work done by the disc per pound is $\frac{w_1 c_1}{g}$, even though this work is not done while the water is passing through the disc, but partly therein and partly in its eye, the vortex being set up and maintained by the friction of the centre of the disc and the inner edges of the vanes. If entry takes place without shock, energy is lost by the sudden change of direction; thus (fig. 124), supposing the direction of a particle before entry to be radial, and its velocity to be Bu_2, then, if the direction of the vane were Bv_3 where $Bv_3 u_2 c_2$ is a parallelogram, then entry would be without shock; but if Bv_2 is the direction of motion at the instant after entry, the actual velocity is Bu where $Bv_2 u c_2$ is a parallelogram. The loss of energy

that takes place is best calculated from the change of velocity relative to the disc, and since the change is from B v_3 to B v_2, the loss of head is $\frac{(v_2\ v_3)^2}{2\,g}$ where $(v_2\ v_3)$ does not mean the product of v_2 and v_3, but the length between the two points so named; but

$$(v_2\ v_3) = (u_2\ v_3) - (u_2\ v_2) = c_2 - u_2 \cot \theta.$$

∴ the loss of head at entry

$$= \mathrm{L}_1 = \frac{1}{2\,g}(c_2 - u_2 \cot \theta)^2 \quad . \quad . \quad . \quad . \quad (1\,c)$$

and it will be avoided if

$$c_2 = u_2 \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2\,c)$$

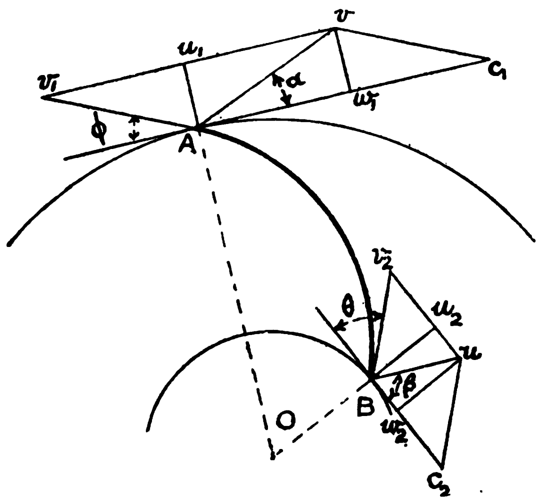


FIG. 123.

Suppose that there is no diffusor, and that the discharge takes place into a volute in which the velocity is v_4, fig. 125, the direction A v_4 being very nearly tangential; then the loss of head is $\frac{(v\ v_4)^2}{2\,g}$, and will be least when $(v\,v_4) = (v\,w_1)$ $= u_1$, or when $v_4 = w_1$, when the loss of head will be

$$\mathrm{L}_2 = \frac{u_1{}^2}{2\,g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\,c)$$

but under these circumstances the final discharge from the pump will take place with a high velocity w_1, and there will be an additional loss, which can best be prevented by using

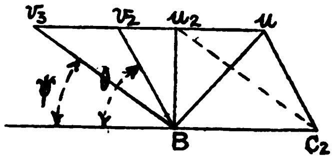


FIG. 124.

a gradually increasing discharge pipe in which the velocity is reduced to some small value D, which may be as low as 2 ft. per second. The additional loss of head is then

$$L_3 = \frac{D_2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4c)$$

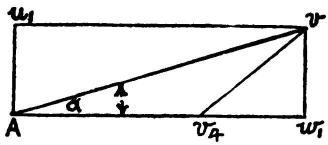


FIG. 125.

If it is impossible to use such a discharge pipe, the losses of energy caused by shock at entry into the volute, and by the waste of the residual energy of discharge, will be

$$L_4 = \frac{v_4{}^2}{2g} + \frac{(v\,v_4)^2}{2g}$$

$$L_4 = \frac{v_4{}^2}{2g} + \frac{(v_4\,w_1)^2}{2g} + \frac{u_1{}^2}{2g}.$$

This will be least when the first two terms on the right are least—that is, when $(A\,v_4) = (v_4\,w_1)$.

Therefore, when a discharge pipe of increasing diameter cannot be used, there will be the least loss of head when $v_4 = \frac{1}{2}\,w_1$, and

$$\therefore L_4 = \frac{w_1{}^2}{4g} + \frac{u_1{}^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5c)$$

and w_1 may be obtained graphically, or by the equation $w_1 = c_1 - u_1 \cot \phi$.

If, as in the earlier types of pumps, there is no volute, then the whole, or at anyrate a very large part, of the kinetic energy at discharge from the disc will be lost. Supposing all is lost, we may write

$$L_5 = \frac{v^2}{2g} = \frac{w_1 + u_1{}^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6c)$$

for pumps with badly-designed casings.

If there is a diffusor of whirlpool chamber surrounding the disc, its breath being constant, as shown in fig. 120, the water is able to reduce its velocity considerably before entering the volute, and the sudden change of direction that then takes place is a cause of a smaller loss of head, because the velocity is reduced. Suppose, fig. 126, a particle of

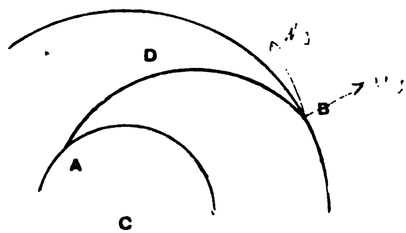


FIG. 126.

water flowing from A to B, the former being the point at which it leaves the disc and the latter where it enters the volute. Let C B = r_3, and u_3 w_3 be the radial and tangential components of the velocity of B; then, since the moment of the external forces about C upon the particle during its passage between A and B equal increase of the momentum from A to B, therefore this latter quantity is zero, because there are no external forces between A and B.

$$\therefore w_1 r_1 - w_3 r_3 = O$$

$$\therefore \frac{w_3}{w_1} = \frac{r_1}{r_3}$$

and

$$\frac{u_3}{u_1} = \frac{r_1}{r_3}$$

since there is continuity of flow.

Therefore the direction of motion of the particle at B has the same inclination to the tangent at B as its direction at

A has to the tangent at A, namely, α. It is clear, then, that A D B is an equiangular spiral.

The larger the ratio $\frac{r_3}{r_1}$ the higher the efficiency of the pump, other things remaining the same, but on the other hand its weight is increased.

If we reason in exactly the same way as before, we shall see that the velocity in the volute must be w_3 if a discharge pipe of increasing diameter may be used, and $\frac{1}{2} w_3$ if not. In the former case the loss of head is

$$L_6 = \frac{u_3^2}{2g} = \frac{u_1^2}{2g}\left(\frac{r_1}{r_3}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7c)$$

and in the latter

$$L_7 = \frac{w_3^2}{4g} + \frac{u_3^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8c)$$

$$= \left(\frac{w_1^2}{4g} + \frac{u_1^2}{2g}\right)\left(\frac{r_1}{r_3}\right)^2$$

We shall now take three cases: firstly of a pump with no special provision to utilise the energy of the water after its discharge from the disc—*i.e.*, with no volute, or a very badly-designed one; secondly, of one with a well-designed volute, but no diffusor; and thirdly, of a pump with both diffusor and volute.

Case I.—Pump with no volute, or a very badly designed one.

In this case, neglecting friction (and this we are compelled to do in the absence of published experiments from which to estimate this quantity), we have—

$$\text{Head} + \text{losses} = \text{work done by disc.}$$

$$H + L_5 = \frac{w_1 c_1}{g},$$

supposing that entry takes place without shock, the value of θ being chosen, so that

$$c_2 = u_2 \cot \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2c)$$

$$H + \frac{w_1}{2g} + \frac{u^2}{2g} = \frac{c_1 w_1}{g}$$

We shall take $u = \frac{1}{4}\sqrt{2gH}$, not because we find that it is so, or that there can be any such rule, but because we wish to make a comparison with the results of Professor Unwin's paper on "Centrifugal Pumps," published in vol. liii. of the Minutes of the Proceedings of the Institution of Civil

Engineers." In writing the present theory, we have obtained much assistance from this paper, but there are some incorrect numerical results that we think should be corrected, as no doubt it has been more widely studied than any other treatise on this subject.

Now, $$w_1 = c_1 - u_1 \cot \phi;$$

$$\therefore 2gH + (c_1 - u_1 \cot \phi)^2 + u_1{}^2 = 2c_1(c_1 - u_1 \cot \phi);$$

$$\therefore 2gH + (c_1{}^2 + u_1{}^2 \cot^2 \phi - 2u_1 c_1 \cot \phi) + u^2{}_1$$
$$= 2c_1{}^2 - 2c_1 u_1 \cot \phi;$$

$$\therefore 2gH = c_1{}^2 - u_1{}^2 \operatorname{cosec}^2 \phi$$

$$c_1 = \sqrt{2gH\left(1 + \frac{\operatorname{cosec}^2 \phi}{16}\right)} \quad . \quad . \quad . \quad (9c)$$

The hydraulic efficiency—

$$\eta = \frac{gH}{c_1 w_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10c)$$

$$= \frac{\text{useful work done per pound}}{\text{total work done by disc per pound}}$$

whence we obtain the following table :—

ϕ		η		c_1
90°	....	·47	....	1·03 $\sqrt{2gH}$
45°	....	·58	....	1·06
30°	....	·65	....	1·12
20°	....	·73	....	1·24
10°	....	·84	....	1·75

In *Engineering*, vol. xxxvii., page 138, will be found the results of some experiments upon a vertical spindle centrifugal pump of the kind we are considering. Its efficiency was very low, even allowing for engine friction, as it never rose above ·41, and the value of $\frac{\text{W.H.P.}}{\text{I.H.P.}}$ was never more than ·3, while the circumferential velocity c_1 was between 1·3 and 1·84 $\sqrt{2gH}$. The value of ϕ is not given, but it is said to be large. The values of η in the table would be lowered by hydraulic friction in practice, and the value of c_1 raised. Also, if entry did not take place without shock, this would also decrease η and increase c_1. What first brought the back-curved vane into use was the fact that the design of the volute was not understood, and, of course, with small values of ϕ the loss L_5 becomes less; we shall show by an actual experiment that, where the casing

is fairly well designed, the efficieney is not so low, even when $\phi = 90$ deg.; but even with a well-defined volute it is often better to have a small value of ϕ—say about 15 deg.—for other reasons besides the efficiency.

CASE II., *a.*—Pump with volute, but no discharge pipe of increasing diameter.

Here again—

$$\text{Head} + \text{losses} = \text{work done by disc.}$$

$$H + L_4 = \frac{c_1 w}{g}.$$

$$H + \frac{w_1^2}{4g} + \frac{u_1^2}{2g} = \frac{c_1 w_1}{g},$$

and
$$w_1 = c_1 - u_1 \cot \phi.$$

$$\therefore 2 c_1 (c_1 - u_1 \cot \phi) - \tfrac{1}{2} (c_1 - u_1 \cot \phi)^2 - u_1^2 = 2 g H.$$

$$\tfrac{3}{2} c_1^2 - c_1 u_1 \cot \phi - u_1^2 (1 + \tfrac{1}{2} \cot^2 \phi) = 2 g H,$$

and
$$\eta = \frac{g H}{c_1 (c_1 - u_1 \cot \phi)}$$

$$\frac{c_1}{\sqrt{2 g H}} = \frac{\cot \phi}{12} + \frac{\sqrt{\frac{1}{16} \cot^2 \phi + 6 (1 + \frac{1}{16} \{ 1 + \cot^2 \phi \})}}{3}$$

putting
$$u_1 = \tfrac{1}{4} \sqrt{2 g H}.$$

From the above equation, which is the same as that obtained by Prof. Unwin, we have calculated the following table, which differs considerably from the results obtained by him in the paper above referred to.

ϕ		η		c_1
90°	....	·725	....	·83 $\sqrt{2 g H}$
45°	....	·77	....	·94
30°	....	·80	....	1·03
20°	....	·84	....	1·189
15°	....	·87	....	1·355

The table given by Professor Unwin is as follows:—

ϕ		η		c_1
90°	....	·6	....	·83 $\sqrt{2 g H}$
45°	....	·8	....	·93
30°	....	·9	....	·98
15°	....	·87	....	1·2

and he, therefore, states that $\phi = 30°$ gives maximum efficiency, whereas the efficiency increases as ϕ decreases.

The reason for the increase of efficiency is that w_1 decreases when ϕ decreases, and L_4 decreases when w_1 decreases; hence the results of the above table.

Case II., *b*.—Pump with volute and discharge pipe of gradually increasing diameter. In this case the velocity in the volute is w_1 and the velocity at discharge is D, so that

$$H + L_2 + L_3 = \frac{c_1 w_1}{g};$$

$$2gH + u_1^2 + D^2 = 2c_1 w_1;$$

$$\text{and } u_1 = \tfrac{1}{4}\sqrt{2gH}.$$

The value of D may be found as low as 2 ft. per second. We have taken it as 7 ft. per second in what follows:—

$$\eta = \frac{H}{H + L_2 + L_3}$$

$$= \frac{H}{H + \frac{H}{16} + k\,H} = \frac{1}{1{\cdot}062 + k}$$

$$\text{where } D = \sqrt{2gHk} = 7.$$

Values of η, k, H, are given in the following table:—

H	k	η
5	·153	·82
8	·096	·86
16	·048	·90
24	·032	·91
32	·024	·92

This is independent, it seems, of ϕ, but in practice it would not be so, because the more gradual the change of velocity of the stream while passing through the disc, the less will be the loss of energy, and the same advantage will be obtained with a small curvature of path. When ϕ is 90 deg., the velocity of whirl is equal to c_1, and the change of speed in the disc is very sudden, but when ϕ is small w_1 decreases, because

$$\frac{c_1 w_1}{g} = \frac{H}{\eta};$$

and even supposing η as great when $\phi = 90$ deg. as when ϕ is small, the product $c_1 w_1$ is constant, and c_1 increases as ϕ decreases, and therefore w_1 diminishes.

Since $$w_1 = c_1 - u_1 \cot \phi$$
and $$u_1 = \tfrac{1}{4}\sqrt{2\,g\,H}$$
$$\therefore c_1 \left(c_1 - \tfrac{1}{4}\sqrt{2\,g\,H}\cot\phi\right) = \frac{g\,H}{\eta}$$
$$= g\,H\,(1{\cdot}062 + k)$$

From the above quadratic the following results are calculated :—

ϕ	= 90°	45°	30°	15°		
$\frac{c_1}{\sqrt{2\,g\,H}}$	= ·745	·88	·991	1·346	when η = ·9 ;	H = 16
	= ·78	·915	1·026	1·376	when η = ·82 ;	H = 5

Comparing this with the previous case, in which there was no discharge pipe of increasing diameter, we see that the speed of the disc is decreased, except when H = 5 and ϕ = 15 deg. The reason for this decrease is that the head lost by shock at discharge from the disc is much reduced by making the velocity in the volute equal to w_1, and saving the kinetic energy $\frac{w_1^{\,2}}{2\,g}$ by converting it into pressure energy in the discharge pipe.

It is interesting to see why, when H = 5 and ϕ = 15 deg., there should be an increase of speed instead of a decrease, as in all other cases. Of course, it is because η is reduced from ·87 to ·82, and this reduction has taken place because L, the loss of head, without a discharge pipe of increasing diameter, is less than $L_2 + L_3$, the loss with this discharge pipe.

To show this numerically—

$$L_2 + L_3 - L_4 = \frac{2\,D^2 - w_1^{\,2}}{4\,g}$$

where w_1 refers to the velocity of whirl, when no gradually increasing discharge pipe is used.

But $$c_1\,w_1 = \frac{g\,H}{\eta}$$
$$w_1 = \frac{2\,g\,H}{2\,\eta \times c_1} = \frac{8{\cdot}02\sqrt{5}}{1{\cdot}74 \times 1{\cdot}355} = 7{\cdot}6\,;$$
$$\therefore L_2 + L_3 - L_4 = \frac{2 \times 7^2 - (7{\cdot}6)^2}{4\,g}$$

and obviously $L_2 + L_3$ is greater than L_4.

With a reduction of D the efficiency would increase, and a discharge pipe would be an advantage theoretically, but to so small an extent as to make its use in practice unadvisable. For values of H, between 16 and 32, it is an undoubted advantage, giving a gain of from 3 to 5 per cent when $\phi = 15$ deg., and experiment might show—we say "might," because no such experiments have been made—that the speed of the disc might be reduced by making $\phi = 30$ deg. to 45 deg. without loss of efficiency. We have the following experiment, however, to support our belief. In *Engineering*, vol. xliii., page 93, there is given a description of centrifugal pumps at Khatatbeh, Egypt. In this case $\phi = 90$ deg., $c_1 = 20{\cdot}8$, $H = 10$, so that $c_1 = {\cdot}82 \sqrt{2 g H}$, $D = 2$, $u_1 = 2{\cdot}7$, allowing a coefficient of contraction of ·9, and v_4, the velocity of the volute, is 12·7, while the mechanical efficiency of pumps and engines is 65 per cent. If we divide this last value by ·9, to allow for engine and shaft friction, we have a hydraulic efficiency of 72¼ per cent, and

$$\eta = \frac{g\,H}{c_1\,w_1} = \frac{g\,H}{c_1^{\,2}}, \text{ because } \phi = 90 \text{ deg.}$$

$$= \frac{32{\cdot}2 \times 10}{(20{\cdot}8)^2} = {\cdot}745.$$

Supposing that inflow took place without shock, the loss of head accounted for by our theory is

$$L = \frac{D_2}{2\,g} + \frac{u_1^{\,2}}{2\,g} + \frac{(c_1 - v_4)^2}{2\,g} = 1{\cdot}211.$$

The friction of the discharge passage would, we estimate, increase this to 1·237 at least. We have disregarded friction in the pump, loss by sudden change of speed, and curvature of path in the fan, and a probable loss of energy at inflow; the additional loss of head, viz., 2·178 ft., which would make the efficiency $\frac{10}{13{\cdot}415} = {\cdot}745$, is most likely due to these causes. The experiment, however, shows that even with an imperfectly-designed volute, it is possible to obtain a very fair efficiency with $\phi = 90$ deg., and such a low lift as 10 ft.

CASE III.—Centrifugal pump, with diffusor or whirlpool chamber, but no discharge pipe of increasing diameter.

Here, again,

$$H + L_7 = \frac{c_1\,w_1}{g}$$

Let $\frac{r_1}{r_3} = m$

then $w_3 = m\, w_1$

$u_3 = m\, u_1$

and $L_7 = \frac{w_3^2}{4g} + \frac{u_3^2}{2g} = m^2 \left\{ \frac{w_1^2}{4g} + \frac{u_1^2}{2g} \right\}$

$\therefore 2\, c_1\, w_1 = 2\, g\, H + m^2\, (u_1^2 + \tfrac{1}{2} w_1^2)$

and $u_1 = \tfrac{1}{4} \sqrt{2\, g\, H}$

$$2\, c_1\, (c_1 - u_1 \cot \phi) = 2\, g\, H \left(1 + \frac{m^2}{16}\right) + \frac{m^2}{2} (c_1^2 + u_1^2 \cot^2 \phi - 2\, c_1\, u_1 \cot \phi)$$

$$c_1^2 \left(2 - \frac{m^2}{2}\right) - c_1\, u_1 \cot \phi\, (2 - m^2) - 2\, g\, H \left(1 + \frac{m^2}{16} + \frac{m^2}{32} \cot^2 \phi\right) = 0$$

$$\frac{c_1}{\sqrt{2\, g\, H}} = \frac{(2 - m^2) \cot \phi}{16 - 4\, m^2} + \frac{\sqrt{(2 - m^2)^2 \cot^2 \phi + (4 - m^2)(32 + 2\, m^2 + m^2 \cot^2 \phi)}}{16 - 4\, m^2}$$

$$\eta = \frac{g\, H}{c_1\, (c_1 - u_1 \cot \phi)}.$$

The following tables are calculated from the above :—

	$m = \frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$
$\phi = 90°$ $c_1 =$	·787 $\sqrt{2\, g\, H}$	·76 $\sqrt{2\, g\, H}$	·745 $\sqrt{2\, g\, H}$	·736 $\sqrt{2\, g\, H}$
45°	·903	·885	·87	·863
30°	1·001	·989	·979	·97
15°	1·33			
$\phi = 90°$ $\eta =$	·806	·86	·90	·92
45°	·846	·89	·925	·945
30°	·88	·905	·933	·952
15°	·94			

We may at once say that no such values of η could be obtained in practice, because hydraulic friction would reduce them, but we believe that there is still some room for improvement in practice, as makers do not guarantee a higher efficiency than ·7, which, allowing for engine friction, would mean a hydraulic efficiency of about 80 per cent.

Comparing the above table with the last but one, we see that when $m = \frac{4}{5}$ there is an increase of about ·07 to ·08 in the efficiency for equal values of ϕ, and that the speed is also decreased. The decrease of m does not bring with it a corresponding decrease of η, and it is evident that, while a small diffusor is an advantage, a large one is hardly worth the additional first cost. We know of no type of pump with such a low value of m as $\frac{1}{2}$. In figs. 117 and 118, $m = \frac{4}{5}$. One side of this diffusor forms a flange to which the side cover is bolted. As in practice such pumps are made with a value of ϕ about 15 deg., and as we know of no experiments in which ϕ is as large as 30 deg., we cannot speak with certainty, but it appears from the above calculations that a larger value of ϕ than 15 deg. might be used with advantage.

It is not worth while considering the case in which a discharge pipe of increasing diameter is used when the pump has a diffusor, as the object of both of these arrangements is to reduce the velocity of flow, and so increase the pressure, and when one is used the other is not required.

We must next consider what occurs when shock takes place at entry. We shall suppose the direction of motion just before entry to be radial.

$$\text{Then H + losses of head} = \frac{c_1 w_1}{g}$$

$$H = \frac{c_1 w_1}{g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} - \frac{(w_1 - v_4)^2}{2g} - \frac{u_1^2}{2g} - \frac{D^2}{2g}$$

supposing there is no whirlpool chamber. The second term on the right-hand side is the loss caused by shock at entry, and the last three are the losses in volute and at discharge.

The hydraulic efficiency is

$$\eta = \frac{gH}{c_1 w_1} = \frac{H}{H + \text{losses}}.$$

If hydraulic friction is taken into account, we should have to subtract a term

$$\frac{F u_1^2}{2g}$$

from the right-hand side of the equation last but one, but as there are no means of calculating the values of F from experiment, we have omitted this term. In the case of the turbine ample information is obtainable; the reverse holds for the centrifugal pump, as makers invariably refuse to

give any information, and to our knowledge there is only one set of experiments in which all necessary particulars are given to compare theory with practice. These will be referred to later on.

Method of calculating the velocities u_1, u_2, v_4, &c.

Here again we are met with a difficulty. Are we to suppose every particle of water to follow a curve, the same as that of a vane? If so, we should have the equations—

$$v_1 = \frac{Q}{(2\pi r_1 b_1 \sin\phi - n b_1 t_1) K}$$

$$u_1 = v_1 \sin\phi = \frac{Q}{(2\pi r_1 b_1 - n b_1 t_1 \operatorname{cosec}\phi) K}$$

and $$u_2 = \frac{Q}{(2\pi r_2 b_2 - n b_2 t_2 \operatorname{cosec}\theta) K}$$

where b_1 b_2 are the breadths of the disc parallel to the axis, t_1 t_2 are the thicknesses of the vanes at their ends, and K is a coefficient of contraction.

On the other hand, are the areas of the passages to be measured as described for turbines in a former page, and illustrated in fig. 118? Want of information compels us to adopt the former method. We have also assumed K as ·9, as we find it gives a better agreement between theory and practice than K = 1, and we have also found it suitable for radial turbines.

CHAPTER XXV.

Comparisons between Theory and Experiment.

That an increase of ϕ reduces the speed we have ample proof; for the pumps at Khatatbeh $c_1 = ·82 \sqrt{2gH}$, and $\phi = 90$ deg. An experiment with a Gwynne Invincible, in 1881, near Amsterdam, gave $c_1 = 1·296 \sqrt{2gH}$, with $\phi = 17$ deg. Mr. Parsons' experiments (vol. xlvii., Minutes of Proceedings of Institute of Civil Engineers) give c_1 at varying values, but always above $\sqrt{2gH}$.

These last are the only experiments about which sufficient details are given to enable us to make a comparison between theory and practice. The dimensions of the pump are only obtainable from a paper subsequently written by Professor Unwin, on the Theory of the Centrifugal Pump (vol. liii.)

In Table A will be seen the results of twelve experiments, and in Table B the results of calculations from the data in Table A. In Table B,

$$\eta_1 = \frac{g\,\mathrm{H}}{c_1\,w_1} = 100,$$

$$\eta_2 = \frac{\mathrm{H}}{\mathrm{H} + \mathrm{L}} \times 100,$$

where L are the losses of energy at inflow and discharge from disc, and at discharge from spiral casing.

Table A.

No. of experiments.	Gallons per minute.	Lift in feet.	Foot-pounds raised per minute.	Foot-pounds indicated per minute.	Revs. per minute.	Efficiency per cent.	Corrected efficiency per cent. η_3.
1	1,012	14·67	148,461	298,438	392	49·74	58·57
4	1,280	14·70	188,160	343,754	398	54·74	62·99
6	1,431	14·75	211,073	374,954	400	56·20	63·95
8	1,568	14·75	231,280	404,737	403	57·01	64·29
10	1,695	14·75	251,987	419,790	405	60·17	67·18
11	1,753	14·80	259,450	435,630	406	59·42	66·39
12	1,012	17·40	176,088	370,458	424	47·53	54·06
15	1,280	17·30	221,440	417,214	428	53·08	59·51
17	1,431	17·40	248,994	447,552	431	53·63	61·86
19	1,568	17·40	272,832	471,552	433	57·86	63·95
21	1,695	17·60	298,310	486,050	435	61·37	67·64
22	1,753	17·60	308,528	494,210	436	62·43	68·68

η_3 is the corrected efficiency per cent from the last column in Table A. It will be seen that some of the experiments have been omitted. This is because we have not calculated them, not because the agreement is better with those we have selected.

In his paper, Professor Unwin gives $r_1 = 9{\cdot}25$ in. $= 2\,r_2$, $b_1 = b_2 = 5{\cdot}75$ in. As he gives no vane thickness, it is probable he neglects them, especially as he assumes that $u_2 = 2\,u_1$, $v_4 = 3\,u_1$, neglecting the vanes. We have taken

$t = \frac{1}{4}$ in., and there are eight vanes, which gives $v_4 = 2{\cdot}35\,u_1$, and $u_2 = 1{\cdot}94\,u_1$, $\phi = 15$ deg., and $\theta = 40$ deg., so that cot $\phi = 3{\cdot}73$, and cot $\theta = 1{\cdot}191$.

The method of calculation is as follows: Let G be the number of gallons per minute, then

$$u_1 = \frac{G}{60 \times 6{\cdot}25\,(2\,\pi\,r_1\,b_1 - n\,b_1\,t_1 \operatorname{cosec}\phi)\,K}$$

$$u_2 = \frac{G}{60 \times 6{\cdot}25\,(2\,\pi\,r_2\,b_2 - n\,b_2\,t_2 \operatorname{cosec}\theta)\,K}$$

Putting K = ·9, and using the values above given, $u_1 = G \times {\cdot}001472$.

Taking experiment (1) as an illustration,

$$G = 1012,$$

and
$$\therefore u_1 = 1{\cdot}48,$$

$$v_4 = 2{\cdot}35\,u_1 = 3{\cdot}48,$$

$$c_1 = 2\,\pi\,r_1 \times \frac{N}{60}$$

where
$$N = \text{revolutions per minute}$$

$$c_1 = 2\,\pi \times \frac{9{\cdot}25}{12} \times \frac{392}{60} = 31{\cdot}55,$$

$$w_1 = c_1 - u_1 \cot\phi = 26{\cdot}04,$$

$$\eta_1 = \frac{g\,H}{c_1\,w_1} \times 100 = \frac{32{\cdot}2 \times 14{\cdot}67}{31{\cdot}55 \times 26{\cdot}04} = 57{\cdot}5,$$

$$\frac{(w_1 - v_4)^2}{2\,g} = 7{\cdot}8\,;\ \frac{u_1{}^2}{2\,g} = {\cdot}034.$$

Let h_4 = loss by shock at entry to volute

$$= \frac{(w_1 - v_4)^2}{2\,g} + \frac{u_1{}^2}{2\,g} = 7{\cdot}834.$$

Let h_3 = loss by shock when entering disc

$$= \frac{(c_2 - u_2 \cot\theta)^2}{2\,g}$$

$$c_2 = \tfrac{1}{2}\,c_1 = 15{\cdot}775,\ u_2 = 1{\cdot}94\,u_1\ \therefore h_3 = 2{\cdot}375.$$

There is also a loss $h_5 = \dfrac{v_4{}^2}{2\,g} = {\cdot}189$ at discharge.

$$\eta_2 = \frac{100\,\mathrm{H}}{\mathrm{H}+\mathrm{L}}$$

$$= \frac{100 \times 14{\cdot}67}{14{\cdot}67 + 7{\cdot}834 + 2{\cdot}375 \div {\cdot}189}$$

$$= 100 \times \frac{14{\cdot}67}{25{\cdot}068} = 58{\cdot}5.$$

η_3 was calculated by Mr. Parsons in the following manner: The foot-pounds raised per minute were 148,461, and those indicated 298,438; hence the actual efficiency per cent was

TABLE B.

No. of experiment.	η_1	η_2	η_3
1	57·5	58·5	58·57
4	58·7	—	62·99
6	60	63·75	63·95
8	61·2	—	64·29
10	62·1	67·4	67·18
11	63·25	68·5	66·39
12	56·2	54·25	54·06
15	58·7	—	59·51
17	60	62	61·86
19	61·2	—	63·97
21	62·6	66·5	67·64
22	63·5	67	68·68

49·74; but this includes friction of the strap by which the pump was driven, of the engine, and bearings of the pump. Also the friction of the outside of the disc is not included in the above theory, and must therefore be subtracted from the indicated power. By experiments afterwards made these losses were found to be about 45,000 foot-pounds. Hence corrected efficiency was

$$\frac{148461}{298438 - 45000} = \frac{148461}{253438}$$

$$= 58{\cdot}57 \text{ per cent} = \eta_3.$$

The agreement between η_1, η_2, η_3 is sufficiently close to support the truth of the above theory. Table C was previously calculated upon the supposition that K = 1, and the thickness of the vanes were neglected

TABLE C.

No. of experiment.	η_1	η_2	η_3	Actual efficiency.
1	55·0	56·0	58·57	49·74
11	58·1	64·0	66·39	59·42
6	56·5	59·9	63·95	56·2
12	54·9	55·1	54·06	54·06
17	56·5	59·2	61·86	53·63
20	58	63·5	65·98	59·79
22	58·7	63·25	68 68	62·43

In table C experiments 11, 20, and 22 give η_1 less than the actual efficiency, while in 6 and 12 the two values are so close that there is not sufficient allowance for friction. It is obvious, then, that K = ·9 gives a closer agreement between theory and practice than K = 1.

CHAPTER XXVI.

CENTRIFUGAL PUMPS AT KHATATBEH, EGYPT.

MESSRS. FARCOT AND CO. have erected at the above station on the Nile five centrifugal pumps, each being driven by a separate engine, and having a vertical shaft connected directly to the engine shaft, which is also vertical. The lift is 10 ft., and the discharge 212 cubic feet per second; the number of revolutions per minute is 32, and the outer diameter is 12·466 ft., giving a circumferential velocity of 20·8 ft. per second, equal to ·82 $\sqrt{2gH}$. The body of each pump, 19 ft. 8 in. in diameter and nearly 12 ft. high, stands on a group of six cast-iron columns A, fig. 127; regulating screws are fitted to each column to adjust the level. The discharge passage, which springs from the volute, is formed first of two cast-iron pipes 27 ft. long, and is extended by a

conduit 33 ft. long. Fig. 127 is a vertical section of one of the pumps. From this it will be seen that the inlet, which is 6 ft. 10·6 in. in diameter at the smallest point, is trumpet-mouthed; it enlarges to 9 ft. 10 in. in diameter, while there is also a curved lip 10 ft. 8 in. in diameter over all. This mouthpiece has a very suitable form to receive the various parts of the current, whether rising horizontal, or falling. In order that the velocity may be gradually increased, there is also an inverted cone B, which swells from a diameter of

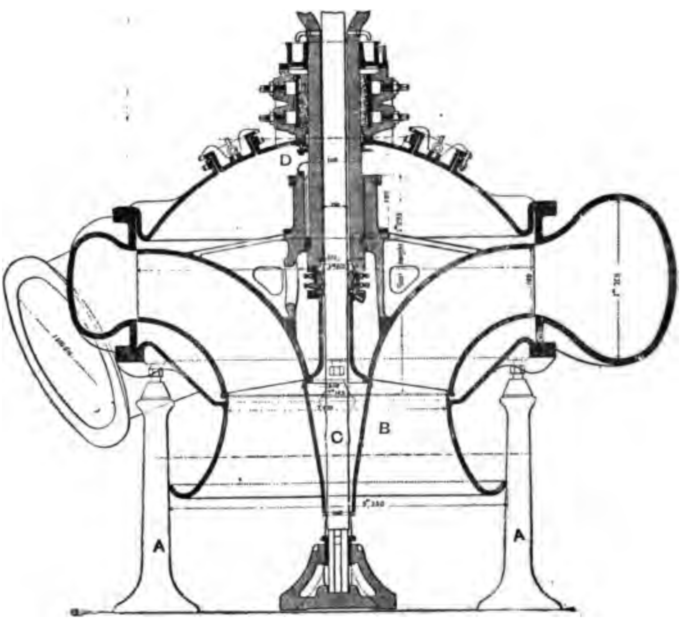


FIG. 127.

11·8 in. to 23·6 in. The annular passage gradually increases the velocity to about 6 ft. per second for the normal discharge of 212 cubic feet per second, when the inlet of the fan is reached. This fan is 4 ft. 8·1 in. high, and the form of the blades is shown by figs. 128 and 129. The capacity available for the water is a solid of revolution, generated by a rotation about the axis of two parabolas concave below,

rising at first almost vertically at their lower part, and then curving outwards almost to the horizontal. They are connected by eight helicoidal vanes, springing from the cone of the lower base of the disc, which are curved spirally backwards through an angle of 60 deg. From this spiral form the vanes gradually change, until they become radial at the circumference of the fan, *i.e.*, $\phi = 90$ deg. The volute is of the usual gradually-increasing section, so that the velocity may be constant therein, but that velocity is not equal to the velocity of whirl as it should be, and, consequently, there is a loss of head at entry into the spiral chamber, as we have already pointed out. The volute is extended by a mouthpiece 5 ft. 3 in. in diameter, fig. 129. To this is attached the discharge conduit above mentioned, which is 50 ft. long, and which changes from a circular to a

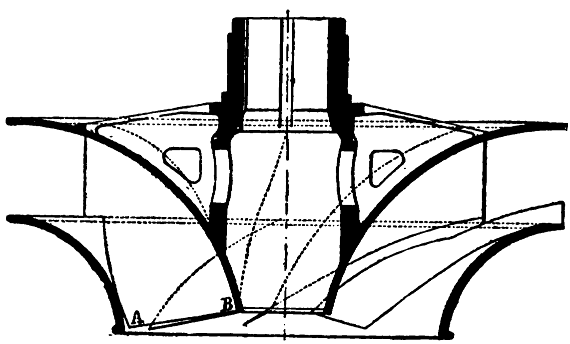


FIG. 128.

rectangular section, $8\frac{1}{4}$ ft. high by 13 ft. wide, so that the velocity of discharge is less than 2 ft. per second. The opening is controlled by a sluice valve.

In order to avoid an immersed footstep, and the consequent difficulties of lubrication, the Fontaine system of turbine pivot is adopted. The lower part of the pump shaft is made hollow, to admit a column or support C, fig. 127, solidly fixed below, and serving as a bearing above the level of the pump itself, and nearly 5 ft. above the highest probable water level. Above the fan, the hollow shaft traversing a stuffing box D in the dome of the pump; this contains a wood bearing, whose adjustment, by screws and lock nuts, is clearly shown. On the top of the fixed iron column A, fig. 134, is fixed a cast-iron cylindrical head,

14·76 in. diameter and 17·72 in. high. This head, on the upper face of which is a bearing of phosphor-bronze (shown to a larger scale in fig. 130), with a concave surface supporting the whole turning load, also serves to centre the hollow shaft, the enlarged cavity of which is at this point provided with a bronze bush B, 10·82 in. deep. Above B, the exterior cast-iron shaft separates into two branches C, rectangular in section and 24 in. apart at their greatest distance, which meet again above, leaving a wide opening between them for

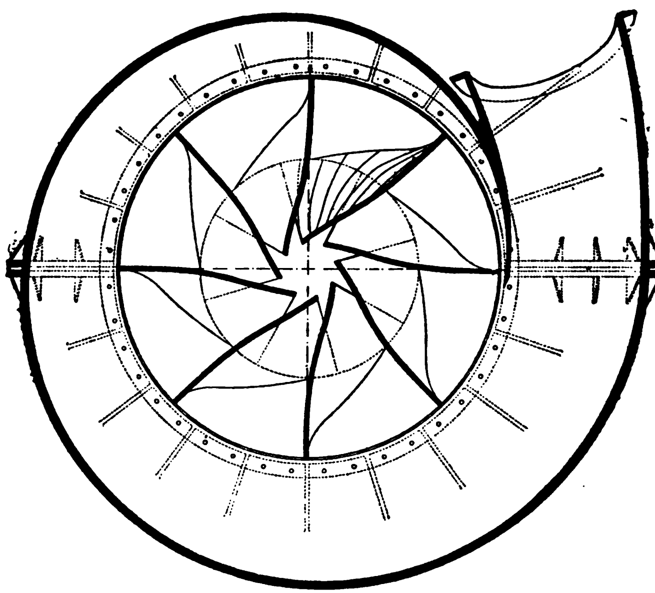
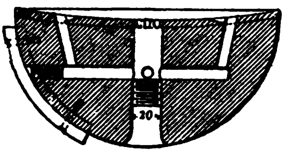


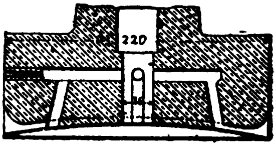
Fig. 129.

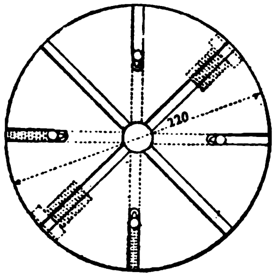
giving access to the pivot. The head of the hollow shaft above this opening is 24·8 in. high, and 21·66 in. in diameter; it is enclosed in a large plummer block bolted to a heavy girder. This cylindrical head is hollow and threaded, so as to form a nut 15·75 in. in diameter, and 21·66 in. deep, in which is screwed the lower end of the engine shaft. Below this point will be seen, in fig 134, the pivot on which the weight of the pump disc and its attachments is borne, the pivot being screwed and fitted with a deep gun-metal lock

nut D, so that it is adjustable vertically as wear takes place; the lower end of the pivot is recessed, and to it is fixed a phosphor bronze bearing (shown to a larger scale in figs. 131 and 132). Between this bearing and the lower fixed bearing,

Fig. 131.







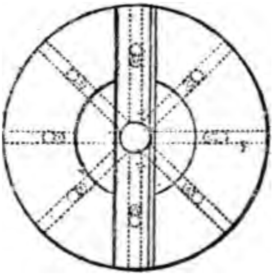


Fig. 130.

Fig. 132.

fig. 130, are placed three loose discs, the upper and lower ones bi-convex, and made of hard steel, and the middle one bi-concave, of phosphor-bronze. These three discs (only one is shown in the figure), which are entirely free, are

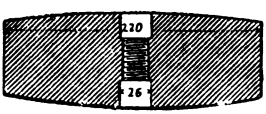


Fig. 133.

intended to distribute between them any inequality of friction, and to divide uniformly the work to be done. In the latter case, the lower disc would have only one-fourth the speed of rotation of the pump, the middle disc one-half, and the upper one three-fourths of its velocity, so that the

relative speed of the four frictional surfaces would be only one-fourth of that of the shaft. These contact surfaces are of such a form as to tend to centre the free discs, but they are also surrounded by a gun-metal sleeve 11·8 in. long. The diameter of the discs is 8·66 in., so that, deducting the surface lost in the oil passages, the weight upon them is 2,270 lb. per square inch. The lubricating arrangements are shown in figs. 130 to 132. On each of the rubbing surfaces

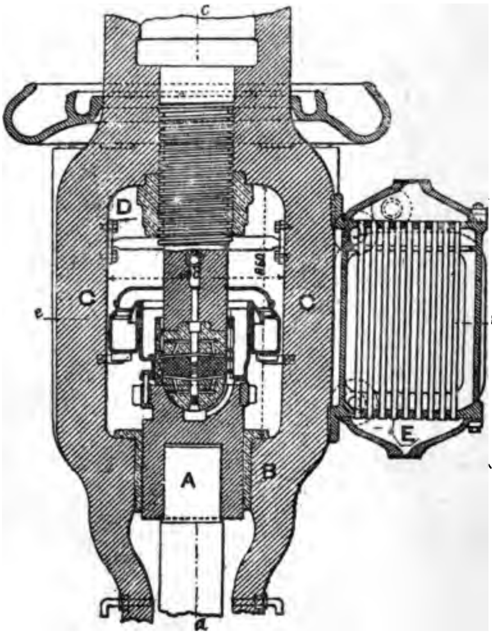


FIG. 134.

four oil chambers are cut, radiating from the centre to the circumference, and four other grooves between the former run from the circumference towards the centre. The sleeve surrounding the discs is pierced with numerous holes. All these parts are contained in a round basin containing oil, fig. 134. Two rotary pumps draw the oil from the basin, and, after passing it through a group of 153 tubes E, deliver

it at the top of the central opening of the pivot. In the box containing the tubes water circulates, in order that the oil may be cooled.

The speed of the engine can be regulated by the governor, which can be adjusted to give from 16 to 42 revolutions per minute. The engine is of the Farcot-Corliss type, with cylinder 39·37 in. diameter, and 5 ft. 10·8 in. stroke. The useful work done by the pump is 65 per cent of the I.H.P., and the consumption per pump horse power hour was guaranteed under 3·85 lb. of good English coal.

This pump clearly shows that it is not necessary to curve back the vanes to obtain a good efficiency.

CHAPTER XXVII.

THE EFFECT OF THE VANE ANGLE ϕ UPON THE DISCHARGE.

FROM what we have previously written, it would appear to be better to make $\phi = 90$ deg., for by this the number of revolutions is reduced in the ratio of 8 to 13, without impairing the efficiency, unless this is somewhat reduced by the greater velocity of whirl required when $\phi = 90$ deg., above what is necessary when $\phi = 15$ deg., which entails a rapid change of velocity and direction of the flow during passage through the disc, and the reader should now know that all such rapid changes are to be avoided. But there is another and more important reason why ϕ should be small under certain circumstances. If the discharge has to be variable, and the pump is required to deliver an amount considerably less than the normal, the vane with a small value of ϕ has a great advantage. It is only possible to explain this mathematically. We have already shown that when there is shock at entry, and friction is neglected—

$$H = \frac{c_1 w_1}{g} - \frac{(c_2 - u_2 \cot \theta)^2}{2g} - \frac{(w_1 - v_4)^2}{2g} - \frac{u_1{}^2 + D^2}{2g}$$

which may be readily thrown into the form—

$$\begin{aligned} 2gH = {} & c_1{}^2\left(1 - \frac{1}{n^2}\right) - u_1{}^2 \operatorname{cosec}^2 \phi - u_2{}^2 \cot^2 \theta \\ & + 2u_2 \frac{c_1}{n} \cot \theta + 2v_4 (c_1 - u_1 \cot \phi) \\ & - v_4{}^2 - D^2 \end{aligned}$$

where $n = \dfrac{r_1}{r_2}$.

This may also be thrown into the form—

$$c_1^2 + k_1 Q c_1 + k_2 Q^2 - 2gH = O,$$

where Q = cubic feet per second, and k_1, k_2 are constants, depending on θ, ϕ, and the dimensions of the pump—that is, k_1, k_2 are constants for any given pump.

Now let us compare two pumps, each of which is required to lift 25 cubic feet per second to a height of 17 ft. Pump A has $\phi = 90$ deg.; pump B has $\phi = 15$ deg. As it is merely a question of comparison, we can neglect friction. Let us first take pump A.

Work done per pound by the disc

$$= H + \frac{D^2}{2g} + \frac{u_1^2}{2g}$$

if the volute is to be so designed that the velocity of whirl is equal to that in the volute.

∴ Work done per pound by disc $= 17 + 1\frac{49}{64}$, if D is 7 ft. per second, and $u_1 = 8$.

$$\eta = \frac{H}{H + \frac{D^2}{2g} + \frac{u_1^2}{2g}} = \cdot906.$$

$$\frac{gH}{w_1 c_1} = \cdot906$$

and $\qquad w_1 = c_1$, since $\phi = 90$ deg.

$$\therefore c_1^2 = \frac{17g}{\cdot906}$$

and $\qquad c_1 = 24\cdot5 = w_1 = v_4,$

Let $\qquad n = 3 \therefore c_2 = 8\cdot17$ nearly.

Let $\qquad u_2 = 10;$

Then $\qquad \cot\theta = \frac{c_2}{u_2} = \cdot817.$

But

$$c_1^2\left(1 + \frac{1}{n^2}\right) + \left(\frac{2u_2\cot\theta}{n} + 2v_4\right)c_1$$

$$- u_1^2\operatorname{cosec}^2\phi - u_2^2\cot^2\theta - v_4^2 - D^2 - 2gH = O;$$

and, putting in the values of θ, ϕ, &c., we may throw this into the form—

$$c_1^2 + 2\cdot45\, Q c_1 - \frac{9}{4} gH - 1\cdot4\, Q^2 = O.$$

Now, if axes O x, O y (fig. 135) be taken, and values of Q be taken as ordinates, and the corresponding values of c_1 from the above equation be taken as abscissæ, we shall obtain a curve B A C which is hyperbola, and which may be traced from the following table :—

Q = 10	......	c_1 = 26·8		
Q = 17	......	c_1 = 24·7		
Q = 25	......	c_1 = 24·5	......	η = ·905
Q = 34	......	c_1 = 26	......	η = ·81
Q = 40	......	c_1 = 27·5	......	η = ·724
Q = 50	......	c_1 = 30·75	......	η = ·578

$$\text{When } Q = O, \; c_1{}^2 = \frac{9}{4} \times 32{\cdot}2\,H = \frac{2n^2}{n^2 - 1}\, g\,H$$

$$c_1 = 35{\cdot}1\,;$$

and, by the above theory, there would be no flow until c_1 reached this value.

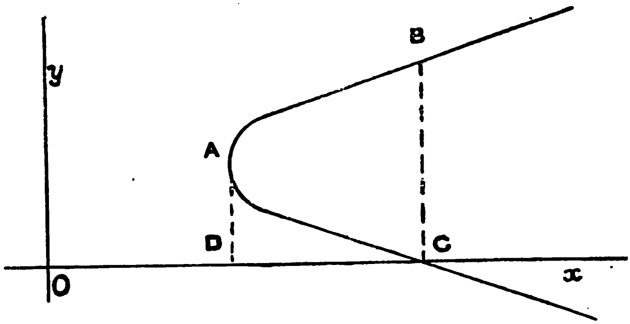


Fig. 135.

In practice, a centrifugal pump will generally begin working at a lower speed than this.

$$c_1 = \sqrt{2\,g\,H}.$$

The above equation for Q and c_1 supposes no rotation to take place within or without the disc when Q = O, and as the inner and outer ends of the vanes set the water near them in motion, this supposition is not true. Hence the want of agreement between theory and practice. When the flow begins, it will increase rapidly until the value of Q is

reached, corresponding to the part of the curve A B, and pumping will continue at a lower velocity than that represented by O C. The flow will cease suddenly if the speed be reduced below O D, and the least discharge will be A D. Increasing the velocity above O D will increase Q, and the discharges represented by the ordinates of A C will never be obtained. Thus, if $\phi = 90$ deg., the discharge can never be less than that which the pump was designed to give in normal working; indeed, in practice the velocity would have to be somewhat greater, or slight changes would suddenly cause the flow to cease. Thus, the values Q = 10 and 17, given in the above table, could never be obtained. Matters will be very different when $\phi = 15$ deg. Let D = 7 and $u_1 = 8$, as before.

$$\eta = \frac{H}{H + \frac{D^2}{2g} + \frac{u_1{}^2}{2g}} = \cdot 906, \text{ as before}$$

$$\eta = \frac{g\,H}{w_1 c_1} = \frac{g\,H}{c_1\,(c_1 - u_1 \cot \phi)}$$

$$\cdot 906 = \frac{17\,g}{c_1\,(c_1 - 8 \cot \phi)}.$$

This is a quadratic for c_1, giving

$$c_1 = 43 \cdot 67.$$

$$v_4 = c_1 - u_1 \cot \phi = 43 \cdot 67 - 29 \cdot 84 = 13 \cdot 83.$$

Let $\quad n = 3$, as before, then $c_2 = \dfrac{43 \cdot 67}{3} = 14 \cdot 55.$

Let $\quad u_2 = 10$, then $\cot \theta = \dfrac{c_2}{u_2} = 1 \cdot 455,$

$\quad \operatorname{cosec}^2 \phi = 14 \cdot 9;$

and substituting these values in the equation

$$c_1{}^2\left(1 - \frac{1}{n^2}\right) + \left(\frac{2\,u_2 \cot \theta}{n} + 2\,v_4\right)c_1$$
$$- u_1{}^2 \operatorname{cosec}^2 \phi - u_2{}^2 \cot^2 \theta - v_4{}^2 - D^2 - 2\,g\,H - O.$$

we obtain

$$c_1{}^2 + 1 \cdot 677\, Q\, c_1 - 2\,g\,H - 4 \cdot 015\, Q^2 = O,$$

where Q = discharge in cubic feet per second.

The above equation gives the following results :—

$Q = 6{\cdot}76$	......	$c_1 = 32{\cdot}3$	......	$\eta = {\cdot}7$
$Q = 10$	......	$c_1 = 32{\cdot}86$	......	$\eta = {\cdot}8$
$Q = 25$	......	$c_1 = 43{\cdot}67$	......	$\eta = {\cdot}905$
$Q = 34$	......	$c_1 = 53{\cdot}5$	......	$\eta = {\cdot}7875$
$Q = 40$	......	$c_1 = 60$	......	$\eta = {\cdot}74$
$Q = 50$	......	$c_1 = 72{\cdot}1$	......	$\eta = {\cdot}61$

The least speed represented by O D, fig. 135, is 32·3 ft. per second, and the discharge may be reduced to 6·76 cubic feet per second. Below this speed there can be no discharge, and the values of Q, represented by the ordinates from A to C, can never be obtained in practice.

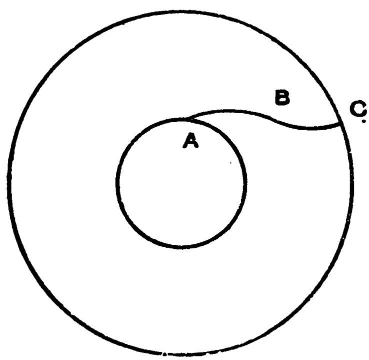


FIG. 136.

Thus if $\phi = 15$ deg., we can reduce the discharge considerably, if necessary ; but if $\phi = 90$ deg., we cannot. For intermediate values of ϕ, the minimum discharge will be greater than 6·76 cubic feet per second, and will approach nearer 25 cubic feet per second, as ϕ approaches 90 deg. Now, if we do not neglect friction, we shall still obtain an equation of the form—

$$c_1^2 + k_1 Q c_1 - 2g H - k_2 Q^2 = O,$$

because, for a given pump, the losses by friction are proportional to Q^2, and the quantity k_2 will be more than it would have been had we neglected friction. The

frictional loss in the fan will be greater when $\phi = 15$ deg. than when $\phi = 90$ deg., but in the volute and discharge pipe the reverse will be the case, the value of Q being the same in both cases.

It is probable, however, that the decrease in the fan will be about balanced by the increase in the volute and discharge pipe, and k_2 will increase and k_1 decrease as ϕ diminishes. We shall still find then that the smaller the value of ϕ the smaller will be the minimum discharge possible. This, we believe, is the only advantage obtained by making ϕ as small as it generally is in practice. It is curious that the very experiments that led to the use of the Appold vane in preference to the Rankine vane, in which $\phi = 90$ deg., show the superiority of the latter, although the experimenter, Mr. Parsons, states at the end of Table IV., page 270, vol. xlvii. of the Minutes of the Institution of Civil Engineers, that the results of these experiments show the advantage of a small value of ϕ. The following are the experiments referred to :—

TABLE II.

Gallons discharged per minute.	Lift in feet.	Foot-pounds of useful work per minute.	Foot-pounds per pound of steam used.	Revolutions per minute.
577	6·500	37,505	8,960	346
746	6·925	51,600	10,809	363
878	6·750	59,265	11,264	368
999	7·085	70,029	12,088	387
1,150	7·750	89,125	13,248	403
1,288	8·333	107,329	15,996	423

Table II. are experiments made with an Appold fan in a volute, and Table IV. are experiments made with a fan having the form of vane recommended by Rankine, and shown in fig. 136.

Thus the efficiency, as measured roughly by the fourth column, is higher in Table IV. than in Table II. for the first four experiments in each case, and had the revolutions been increased above 353 in Table IV., there is little doubt that the Rankine fan would still have been superior to the other. The curve A B, fig. 136, is an involute of a circle, and B C is

any other curve, an arc of a circle suppose, which cuts the outer circumference of the disc at 90 deg. The construction of the involute has already been explained in connection

TABLE IV.

Gallons discharged per minute.	Lift in feet.	Foot-pounds of useful work per minute.	Foot-pounds per pound of steam used.	Revolutions per minute.
580	6·333	36,731	9,675	324
743	6·667	49,528	10,857	334
879	7·000	61,530	11,692	343
996	7·333	73,036	12,954	353

with fig. 58. It is intended, as in the turbine, to lessen contraction.

CHAPTER XXVIII.

ON THE VARIATION OF PRESSURE IN A CENTRIFUGAL PUMP.

LET the axis of rotation be supposed vertical for simplicity of calculation, and let h be the height of the lower surface of the water above the horizontal plane bisecting the fan.

Then if p_2 be the pressure per square foot above the atmosphere at inflow,

$$\frac{p_2}{62 \cdot 5} = h - \frac{u_2^2}{2g}$$

supposing that inflow is radial.

At any radius r in the fan let the absolute velocity be v_5, and let c and w correspond to $c_1\, w_1$ at radius r_1. Then the work done by the fan from radius r_2 to r is $\frac{c\,w}{g}$ per pound, and this is manifested by an increase of pressure $p_3 - p_2$ and a change of velocity from u_2 to v_5.

$$\therefore \frac{c\,w}{g} + h = \frac{p_3}{62 \cdot 5} + \frac{v_5^2}{2g}$$

therefore on leaving the disc the pressure is such that

$$\frac{p_4}{62 \cdot 5} = \frac{c_1 w_1}{g} + h - \frac{v_2}{2g}$$

$$= \frac{c_1 (c_1 - u_1 \cot \phi)}{g} + h - \frac{u_1^2 + w_1^2}{2g}$$

$$= \frac{2c_1^2 - 2c_1 u_1 \cot \phi + 2gh - u_1^2 - c_1^2 - u_1^2 \cot^2 \phi + 2c_1 u_1 \cot \phi}{2g}$$

$$= \frac{1}{2g} \left(c_1^2 - u_1^2 \operatorname{cosec}^2 \phi + 2gh\right)$$

If there is no further gain of head in consequence of a badly-designed casing, then

$$\frac{p_4}{62 \cdot 5} = \mathrm{H} + h = \frac{1}{2g} \left(c_1^2 - u_1^2 \operatorname{cosec}^2 \phi + 2gh\right)$$

$$\mathrm{H} = \frac{1}{2g} \left(c_1^2 - u_1^2 \operatorname{cosec}^2 \phi\right)$$

which will enable us to obtain the equation—

$$c_1 = \sqrt{2g\mathrm{H}} \left(1 + \frac{\operatorname{cosec}^2 \phi}{16}\right) \quad . \quad . \quad . \quad . \quad (9c)$$

by putting $u = \frac{1}{4} \sqrt{2g\mathrm{H}}$.

We have already obtained this equation in a slightly different manner.

Next suppose that $v_4 = w_1$; then there is no increase of pressure by shock in the volute, but in the discharge pipe there is a gain of pressure $p_5 - p_4$, such that

$$\frac{p_5}{62 \cdot 5} = \frac{p_4}{62 \cdot 5} + \frac{v_4^2 - \mathrm{D}^2}{2g} - h_1$$

where $h_1 - h$ is the height above the ower level of the water, at which the velocity is D

$$\therefore \frac{p_5}{62 \cdot 5} = \frac{1}{2g} \left\{c_1^2 - u_1^2 \operatorname{cosec}^2 \phi + 2g(h - h_1) + w_1^2 - \mathrm{D}^2\right\}$$

and by putting $\mathrm{H} + h - h_1 = \frac{p_5}{62 \cdot 5}$, and $w_1 = c_1 - u_1 \cot \phi$,

$$\frac{p_5}{62 \cdot 5} = \frac{1}{2g} \Big\{2c_1^2 - u_1^2 \operatorname{cosec}^2 \phi + u_1^2 \cot^2 \phi$$
$$- 2c_1 u_1 \cot \phi + 2g\left(h - h_1\right) - (\mathrm{D}^2)\Big\}$$

$$\frac{p_5}{62{\cdot}5} = \frac{1}{2g}\left\{2c_1^2 - u_1^2 - 2c_1 u_1 \cot\phi + 2g\left(h - h_1\right) - D^2\right\}$$
$$= H + h - h_1$$
$$\therefore 2gH = 2c_1^2 - u_1^2 - 2c_1 u_1 \cot\phi - D^2$$

so that this is merely another method of arriving at an equation between c_1, u_1, H and ϕ.

Next, let us suppose that $v_4 = \frac{1}{2} w_1$, and that the diameter of the discharge pipe is not increased; then there is a gain

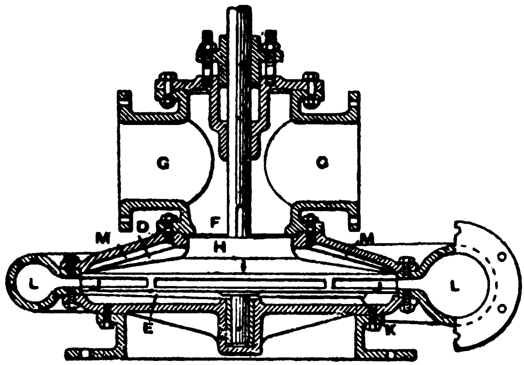


FIG. 137.

of pressure by shock when the water leaves the disc and enters the volute. This gain is

$$\frac{1}{62{\cdot}5}\left(p_6 - p_4\right) = \frac{w_1^2}{4g}$$
$$\therefore \frac{p_6}{62{\cdot}5} = \frac{1}{2g}\left(c_1^2 - u_1^2 \operatorname{cosec}^2\phi + 2gh + \frac{w_1^2}{2}\right)$$
$$= \frac{1}{2g}\left(\frac{3}{2}c_1^2 - \frac{u_1^2}{2}\left[1 + \operatorname{cosec}^2\phi\right] + 2gh - c_1 u_1 \cot\phi\right)$$
$$\text{and } p_6 = 62{\cdot}5\left(H + h\right)$$
$$\therefore 2gH = \frac{3}{2}c_1^2 - \frac{u_1^2}{2}\left[1 + \operatorname{cosec}^2\phi\right] + 2gh - c_1 u_1 \cot\phi$$

It will be noticed that the effect of reducing ϕ is to decrease p_4, p_5, and p_6.

CHAPTER XXIX.

The Balancing of Centrifugal Pumps.

The reader will now perceive that the pressure on the disc may cause a thrust along the shaft which will require balancing. Pumps with two inlets, as in figs. 119 and 120, are self-balancing, but when the inlet is at one side the thrust may be considerable. In a paper on this subject, read by Mr. J. Richards before the Technical Society of the Pacific Coast, two pumps were referred to, and are here shown in figs. 137 and 138, in which special arrangements were made to obtain a balance.

Fig. 137 is a vertical section. The water enters by double inlets G at the top of the pump, and then through the nozzle H to the interior of the wheel I, which is not in section, and is discharged from the periphery J into the casing L. The heads are from 50 ft. to 90 ft., much greater than in this country. On the top of the casing will be noticed baffling vanes M to lessen the rotation of the water in the space D above the disc. There are also vanes K under the disc at E to set the water in rotation. To understand the reason for this construction, let us suppose the sides of the fan to be perfectly smooth outside, so that the water in D and E does not rotate. Then leakage will take place from the volute until the pressure in D and E is the same as that in the volute, causing an upward thrust, because the pressure on H is small. To a certain extent this is an advantage, as it balances the dead weight; but when it exceeds this it causes sufficient thrust, according to Mr. Richards, to do considerable damage. In order, then, to increase the downward pressure the baffling vanes are used, which reduce the rotation above the fan; and to decrease the upward pressure, the water is rotated below by the vanes K; hence the amount of upthrust can be regulated by the amount of water rotation under the wheel at E. This is a very sensitive kind of balance. In one case 1 in. cut from the tips of the vanes K, which were only $\frac{3}{8}$ in. square, made a difference of more than 300 lb. in the thrust upon the shaft. If the space E is much widened by raising the wheel, then the effect of the vanes K is less, and the upthrust increases as the wheel rises, because the rotation is not so rapid in the increased space under the wheel.

Fig. 138 shows a somewhat superior type of pump to that in fig. 137. In all pumps with encased fans a water-tight joint should be maintain around the nipple H. Any water forced back round this nipple will re-enter at F, and merely circulate in the pump. This is an objection to encased wheels, and should prevent them being used for water containing grit or sand. The fan in fig. 138 is not encased; the inlet A is at the side, so as to be accessible and easy to remove. The main casing T is volute in form, and so

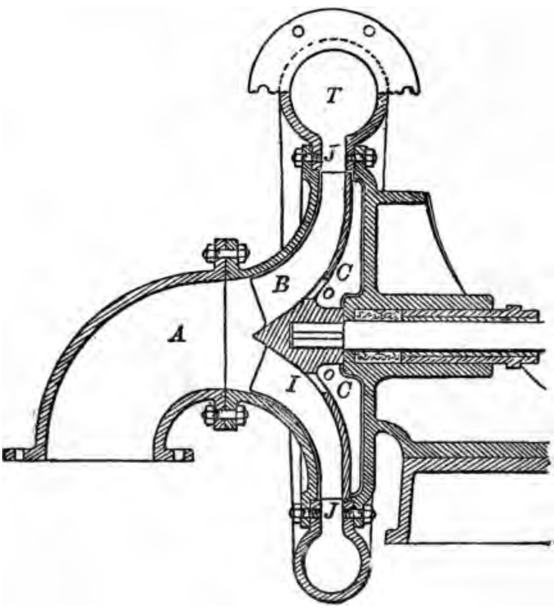


FIG. 138.

constructed as to be set in any position on the frame The course of the water from A through B to J is by easy curves. The packing round the spindle is placed inside the main bearing, and there is, by reason of the concave form, but little overhang of the wheel. The wheel is a disc, with vanes B and I on its front side, and balancing vanes C, which reduce the pressure on the back in the same manner as the vanes K in fig. 137. There is a difference of ¾ in. to

$1\frac{1}{2}$ in. between the diameters of the working and balancing vanes. This is necessary in order to obtain a perfect balance, as we shall presently show mathematically.

It must not be imagined that because a pump has a side inlet that its disc needs balancing vanes. Fig. 139 shows the disc of a pump with horizontal axis. Since it is encased, there is a balance of pressure on the conical portion, and it is only at the centre that there is any danger of end thrust, of which there are two causes. The first is the difference of pressure that would exist on the two sides of the part of the disc connecting the boss of the encased portion. This is obviated by six holes B. The second is the change of flow from an axial to a radial direction, which would give rise to a force less than 70 lb., supposing the velocity of inflow to be 10 ft. per second, the diameter of the inlet being 8 in. This may be calculated as follows: The horizontal momentum of the water is destroyed, and therefore, if this is the cause of

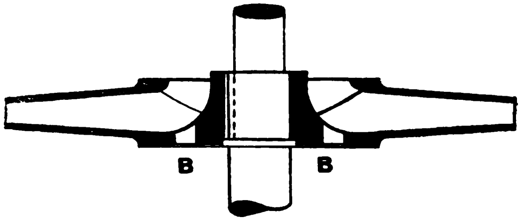


Fig. 139.

a pressure of p pounds per square foot, and A is the area of the inlet in square feet, the impulse of the pressure p A in time t is

$$p\,A\,t = \frac{W\,V}{g},$$

where W = weight of water passing through inlet in t seconds, and V is its velocity.

$$\therefore p\,A\,t = \frac{62{\cdot}5\,A\,V\,t.\,V}{g}$$

$$p\,A = \frac{62{\cdot}5\,A\,V^2}{g}$$

$$= \frac{62{\cdot}5 \times {\cdot}7854 \times (\tfrac{2}{3})^2 \times 100}{32}$$

$$= 68{\cdot}2\,\text{lb.},$$

causing a thrust to the left too small to be of any importance.

We shall now explain mathematically the statements we have made above about balancing vanes. Suppose we have a cylindrical vessel (fig. 140) in which are radial vanes rotating with an angular velocity ω, so that each particle of water has a velocity $r\,\omega$, where r is the radius of the circle in which it moves. Then it is obvious that the pressure must increase from the centre to the circumference; because, if we consider any ring $a\,f\,e$, its motion in a circle would require a centripetal force to prevent it moving outwards. Thus the pressure on the outside is

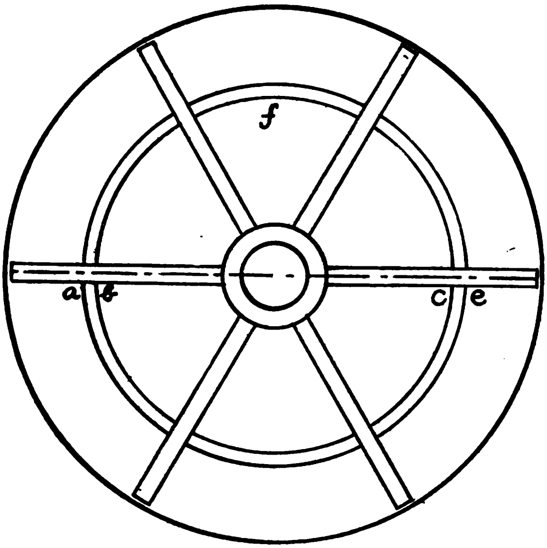


FIG. 140.

greater than that on the inside of the ring. Let the internal radius of the ring be r, and its thickness $d\,r$, so that its mean radius is—

$$r + \frac{d\,r}{2};$$

and suppose we take 1 ft. depth parallel to the axis. The weight of half the ring $a\,b\,f\,c\,e$ is—

$$62{\cdot}5 \times \pi\left(r + \frac{d\,r}{2}\right) d\,r,$$

and the resultant of its centrifugal force is perpendicular to the diameter $a\,e$, and is—

$$F = 125\left(r + \frac{d\,r}{2}\right)\frac{r\,\omega^2}{g}\,d\,r.$$

Let p be the pressure on the inside of the ring, and $p + d\,p$ that on the outside in pounds per square foot; then—

$$p + \frac{d\,p}{2}$$

is the mean pressure on $a\,b$ and $c\,e$ in pounds per square foot. The resultant of these fluid pressures is—

$$P = 2\,(p + d\,p)\,(r + d\,r) - 2\,p\,r - 2\,p\,d\,r,$$

acting in the opposite direction to F. But $F = P$ for equilibrium, and we may neglect small quantities of the second degree.

$$\therefore \frac{2\,r^2\,\omega^2}{g} = \frac{2\,r\,d\,p}{62{\cdot}5};$$

$$\therefore \frac{r\,\omega^2\,d\,r}{g} = \frac{d\,p}{62{\cdot}5}.$$

Integrating—

$$\frac{r^2\,\omega^2}{2\,g} = \frac{p}{62{\cdot}5} + C;$$

$$\frac{V^2}{2\,g} = \frac{p}{62{\cdot}5} + C.$$

Let V_1 be the velocity at the outer radius of the vessel R_1, and let the pressure be p_1 at that radius; then

$$\frac{V_1^{\,2}}{2\,g} = \frac{p_1}{62{\cdot}5} + C;$$

$$\therefore \frac{V_1^{\,2}}{2\,g} = \frac{p_1}{62{\cdot}5} + \frac{V^2}{2\,g} - \frac{p}{62{\cdot}5};$$

$$\frac{p}{62{\cdot}5} = \frac{p_1}{62{\cdot}5} - \frac{1}{2\,g}\left(V_1^{\,2} - V^2\right) = \frac{p_1}{62{\cdot}5} - \frac{w^2}{2\,g}\left(R_1^{\,2} - r^2\right)$$

Hence, the greater ω becomes, the less will p be; also, if R_1 could be decreased by decreasing the length of the vanes, p would increase. We said above that if the vanes K, fig. 137, had 1 in. cut from their tips, a difference of more than 300 lb. in the thrust upon the shaft was produced. This, then, was partly due to the increase of p at each point under the fan, and also because the inner radius of the area upon which the pressure p_1 outside the vortex acted was

decreased. The raising of the fan also increased the upward thrust, because the space under it was increased, and the shallow balancing vanes did not set the water in such rapid rotation, because it would be easily reduced by the friction of the casing, causing the water to leak back past the shallow vanes. Thus ω was decreased, and therefore p increased.

In fig. 138 we have to deal with the difference of pressure on the two sides of the disc. On the left there is the flow, on the right none worth taking into account. We have shown above that if p_4 be the pressure on the left side at radius r_1,

$$\frac{p_4}{62 \cdot 5} = \frac{1}{2g}(c_1{}^2 - u_1{}^2 \operatorname{cosec}^2 \phi + 2gh),$$

and if p_3 be the pressure at radius r in the disc on the same side,

$$\frac{p_3}{62 \cdot 5} = \frac{1}{2g}(c^2 - u^2 \operatorname{cosec}^2 \psi + 2gh),$$

where c, u, and ψ at radius r correspond to c_1, u_1, and ϕ at radius r_1.

Now, on the right side of the disc,

$$\frac{p}{62 \cdot 5} = \frac{p_1}{62 \cdot 5} - \frac{1}{2g}(\mathrm{V}_1{}^2 - \mathrm{V}^2);$$

also $$\mathrm{V} = c, \text{ and } p_1 = p_4;$$

$$\therefore \frac{p_4}{62 \cdot 5} = \frac{p}{62 \cdot 5} + \frac{1}{2g}(\mathrm{V}_1{}^2 - c^2),$$

and

$$\frac{p_4 - p_3}{62 \cdot 5} = \frac{1}{2g}(c_1{}^2 - c^2 - u_1{}^2 \operatorname{cosec}^2 \phi + u^2 \operatorname{cosec}^2 \psi);$$

$$\therefore \frac{p - p_3}{62 \cdot 5} + \frac{1}{2g}\left(\mathrm{V}_1{}^2 - c\right) =$$

$$\frac{1}{2g} c_1{}^2 - c^2 - u_1{}^2 \operatorname{cosec}^2 \phi + u^2 \operatorname{cosec}^2 \psi);$$

$$\therefore \frac{p - p_3}{62 \cdot 5} = \frac{1}{2g}(c_1{}^2 - \mathrm{V}_1{}^2 - u_1{}^2 \operatorname{cosec}^2 \phi + u^2 \operatorname{cosec}^2 \psi).$$

With the usual form of vane curving back, $u_1 \operatorname{cosec} \phi$ is greater than $u \operatorname{cosec} \psi$, and in fig. 138 V_1 is less than c_1. By a proper choice of V_1—that is, of R_1, the radius of the balancing vanes—we can make p greater, equal to, or less than p_3. It should be so arranged that the whole pressure inside a circle of radius R_1 is less on the right of the disc

than on the left—*i.e.*, the average value of p should be less than p_3, for between the radii R_1 and r_1 the pressure is greater on the right than on the left of the disc; that on the right is p_4, and that on the left is less than p_4, except at radius r_1, because it decreases with the radius. By this means a perfect balance can be obtained. The value of R_1 is, of course, best found by experiment. Its calculation would be very complicated, involving the integral calculus, and as there will be a little slip of water past the vanes, which we have neglected in our calculations, theory would not be so accurate as experiment.

CHAPTER XXX.

Method of Designing a Centrifugal Pump.

Fig. 141 shows the variation of efficiency for centrifugal reciprocating pumps. It is taken from a paper on "The Relative Efficiency of Centrifugal and Reciprocating Pumps," by Mr. W. O. Webber, read before the American Society of Mechanical Engineers. The ordinates represent the ratio of the water horse power or useful work done by the pump

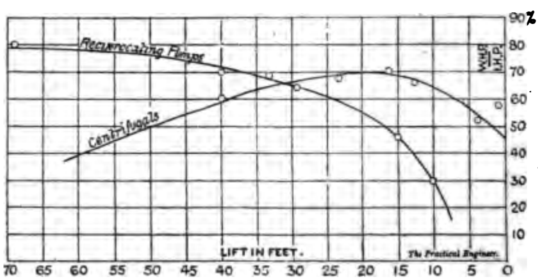


Fig. 141.

to the indicated horse power. The centrifugal curve reaches its highest point for a 20 ft. lift when the efficiency is 70 per cent. This curve, together with the equation,

$$c_1 w_1 = \frac{g\,H}{\eta}$$

may be made use of for the design of a centrifugal pump in the following manner; it will be best to take a numerical example.

EXAMPLE I.

A pump is required to lift 35 cubic feet of water per second to a height of 20 ft. There is to be no discharge pipe of increasing diameter, but a diffusor or whirlpool chamber whose outer diameter is $1\frac{1}{4}$ than of the fan.

In order to obtain a high efficiency under these circumstances ϕ must be small, and is assumed to be 15 deg. Then

$$\cot\phi = 3{\cdot}73, \text{ and } \operatorname{cosec}\phi = 3{\cdot}86.$$

The hydraulic efficiency of η is assumed to be 75 per cent, allowing for the friction of the engine and shafting of the pump. This assumes that a little over 93 per cent of the power of the engine is transferred to the fan. Then,

$$c_1 w_1 = \frac{g\,H}{\eta}$$

$$c_1{}^2 - c_1 u_1 \cot\phi = \frac{g\,H}{\eta}$$

Let $u_1 = 5,\ H = 20,$

$$c_1{}^2 - 18{\cdot}65\,c_1 - 858{\cdot}6 = O$$

$$c_1 = \frac{18{\cdot}65 + \sqrt{348 + 3434{\cdot}4}}{2}$$

$$= \frac{18{\cdot}65 + \sqrt{3782{\cdot}4}}{2} = 40{\cdot}07.$$

Let us suppose the number of revolutions per minute is to be about 140. Then, a diameter of $5\frac{1}{2}$ ft. will give 139·2 revolutions with a velocity of 40·07 ft. per second. Let

$$2\,r_1 = 5\tfrac{1}{2}\text{ ft.}$$

Suppose there are six vanes, and let

$$t = \tfrac{1}{2}\text{ in.} = \tfrac{1}{24}\text{ ft.}$$

Then the breadth b_1 of the fan at radius r_1 is

$$b_1 = \frac{12\,Q}{K\,u_1\,(2\,\pi\,r_1 - n\,t\operatorname{cosec}\phi)}$$

$$= \frac{35 \times 12}{{\cdot}9 \times (17{\cdot}25 - {\cdot}966) \times 5}$$

$$= 5{\cdot}73\text{ in.}$$

$$w_1 = c_1 - u_1\cot\phi = 40{\cdot}07 - 18{\cdot}65 = 21{\cdot}42$$

and since there is a diffusor, whose radius r_3 is $1\frac{1}{4}\, r_1$, and since

$$w_3\, r_3 = w_1\, r_1$$

$$\therefore w_3 = \tfrac{4}{5}\, w_1 = 17{\cdot}136.$$

There is to be no discharge pipe of increasing diameter, and therefore the velocity in the volute is to be $\frac{1}{2}\, w_3$—

$$v_4 = \tfrac{1}{2}\, w_3 = 8{\cdot}568.$$

The diameter of the discharge pipe—

$$d = 12\sqrt{\frac{Q \times 4}{v_4 \times \pi}} = 27{\cdot}4 \text{ in.}$$

The diameter of the suction pipe is generally made equal to the diameter of the discharge pipe; but there is no necessity for this.

Referring to fig. 119, the curve E D should be made approximately in the form of an equiangular spiral, so that the direction of flow may not be suddenly changed at discharge from the fan, and the water may be allowed to flow in the diffusor as theory requires. The proper angle may be obtained by setting out the parallelogram $A\, c_1\, v\, v_1$ (fig. 123).

The diameter at inflow B B depends on the velocity we assume at that point, and this should not differ much from u_2. This latter velocity may be here assumed to be 8 ft. per second—

$$\cot \theta = \frac{c_2}{u_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2c)$$

$$= \frac{c_1\, r_2}{r_1\, u_2}.$$

There is no fixed rule for $\dfrac{r_1}{r_2}$ but we may take it as 3.

$$r_2 = {\cdot}916 \text{ ft.}$$

$$\therefore \cot \theta = \frac{40{\cdot}07}{3} \div 8 = 1{\cdot}672;$$

$$\theta = 30^\circ.\, 53'.\,; \ \operatorname{cosec} \theta = 1{\cdot}948.$$

The breadth of the fan at inflow is—

$$b_2 = \frac{12\, Q}{K\,(2\pi\, r_2 - n\, t \operatorname{cosec} \theta)\, u_2} \text{ inches.}$$

$$= \frac{35 \times 12}{{\cdot}9 \times (5{\cdot}75 - {\cdot}4895) \times {\cdot}8}$$

$$= 11{\cdot}1.$$

At B B an obstruction is caused by the collars pinned on the shaft to prevent lateral motion of the fan. It will first be necessary to calculate the diameter of the shaft. The useful work done per minute by the pump is—

$$35 \times 62{\cdot}5 \times 20 \times 60 \text{ foot-pounds.}$$

Let N be the revolutions per minute = 139·2, and T the mean twisting moment on the shaft in inch-pounds. Then—

$$\frac{2\pi T N}{12} = \text{foot-pounds per minute ;}$$

and if d = shaft diameter,

$$T = {\cdot}196 f d^3,$$

where f is the mean stress in pounds = 9000 ;

$$\therefore d^3 = \frac{35 \times 62{\cdot}5 \times 20 \times 60 \times 12}{{\cdot}7 \times 2\pi \times {\cdot}196 \times 9000 \times 139{\cdot}2},$$

allowing for an actual sufficiency of 70 per cent,

$$d = 3{\cdot}08 \text{ in.}$$

Allowing for keyways, this may be increased to $3\frac{5}{8}$ in., and the collars may be made 6 in. in diameter.

Let A be the area at B B ; then

$$A = \frac{35 \times 144}{8 \times 2} + 28{\cdot}27$$

28·27 being the area of a circle 6 in. diameter, and the velocity or flow 8 ft. per second,

$$A = 343{\cdot}27 \text{ square inches ;}$$

diameter at B = 21 in.

The side passages are ellipses whose axes are 27·4 and 13·7 inches at right angles and parallel to the direction of the shaft; these give a combined area equal to that of the suction pipe if it is made the same diameter as the discharge pipe, viz., 27·4 in. If, however, the velocity at A, figs. 119 and 120, is 8 ft. per second, then the diameter of the suction pipe must be $28\frac{1}{4}$ in. to $28\frac{3}{8}$ in., and the elliptical axes must be $28\frac{1}{4}$ in. to $28\frac{3}{8}$ in. and $14\frac{1}{8}$ in. to $14\frac{3}{16}$ in.

Of course, a gradual decrease of velocity from 8·568 at A to 8 ft. per second at B might be easily managed by a gradual increase of section in the side passages.

Besides hydraulic friction, the loss of head is

$$L_7 = \frac{w_3^2}{4g} + \frac{u_3^2}{2g}$$

$$= \frac{(17{\cdot}136)^2}{4g} + \frac{(5 \times \frac{4}{5})^2}{2g}$$

$$= 1{\cdot}14 + {\cdot}25 = 1{\cdot}39 \text{ ft.}$$

The hydraulic efficiency $= {\cdot}75$

$$= \frac{20}{26{\cdot}666}.$$

Therefore, we have allowed for a loss of head of 5·276 by hydraulic friction.

EXAMPLE II.

The lift is 16 ft., and there are 40 ft. of piping, whose friction must be taken into account. The number of cubic feet per second is 25. A discharge pipe of increasing diameter may be used. It is necessary to assume some additional head in consequence of the pipe friction. To save troublesome calculation, let this be 1 ft., and assume $\phi = 80$ deg., in order that the velocity of the disc may be reduced.

$$\cot \phi = {\cdot}176, \operatorname{cosec} \phi = 1{\cdot}015\,;$$

$$c_1 w_1 = \frac{g\,H}{\eta}.$$

$$c_1^2 - c_1 u_1 \cot \phi = \frac{g\,H}{\eta}.$$

Let $u_1 = 5\,;\ H = 17:\ \eta = {\cdot}75\,;$

$$c_1^2 - c_1 \times {\cdot}88 - 730 = 0\,;$$

$$c_1 = 27{\cdot}5 \text{ nearly.}$$

Supposing inflow to take place from both sides, as in fig. 99, and neglecting the obstruction of the passages caused by the shaft and collars, we find—

$$A = \frac{25 \times 144}{8 \times 2} = 225 \text{ square inches.}$$

The diameter at B is therefore about 17 in. approximately. We may therefore make $2\,r_2$ about 18 in., and if $2\,r_1$ is 54 in., then—

$$\frac{r_1}{r_2} = 3, \text{ as in Example I.}$$

The number of revolutions per minute is obtained from the equation—

$$60\,c_1 = 2\,\pi\,r_1\,\mathrm{N}$$

$$\mathrm{N} = \frac{60 \times 27{\cdot}5 \times 12}{54\,\pi}$$

$$= 117$$

$$b_1 = \frac{12\,\mathrm{Q}}{\mathrm{K}\,u_1\,(2\,\pi\,r_1 - n\,t\,\mathrm{cosec}\,\phi)} \text{ inches}$$

$$= \frac{12 \times 25}{{\cdot}9 \times 5\left(\frac{\pi \times 54}{12} - 6 \times \frac{1}{24} \times 1{\cdot}015\right)}$$

$$= 4{\cdot}84 \text{ inches}$$

$$c_2 = c_1 \times \frac{r_2}{r_1} = \frac{27{\cdot}5}{3} = 9{\cdot}16.$$

Let $u_2 = 8$; then $\cot\theta = \frac{c_2}{u_2} = 1{\cdot}145$

$$\theta = 41^\circ\ 8'; \ \mathrm{cosec}\,\theta = 1{\cdot}52$$

$$b_2 = \frac{12\,\mathrm{Q}}{\mathrm{K}\,u_2\,(2\,\pi\,r_2 - n\,t\,\mathrm{cosec}\,\theta)} \text{ inches}$$

$$= \frac{12 \times 25}{{\cdot}9 \times 8\left(\pi \times \frac{18}{12} - 6 \times \frac{1}{24} \times 1{\cdot}52\right)}$$

$$= 9{\cdot}64 \text{ inches.}$$

The shaft diameter can be calculated as in Example I., and the passages at B B increased by the necessary amount. An increase in diameter from 17 in. to 18 in. would add about 27 square inches area, which would allow for an obstruction in the passage of $5\frac{7}{8}$ in. in diameter.

$$w_1 = c_1 = u_1 \cot\phi$$

$$= 27{\cdot}5 - {\cdot}88 = 26{\cdot}62.$$

This is the velocity in the volute, since there is no diffusor. Let A_1 be the area of the volute at discharge—

$$\mathrm{A}_1 = \frac{25 \times 144}{26{\cdot}62} = 135 \text{ square inches.}$$

Diameter at discharge $= 13\frac{1}{8}$ in.

Let A_2 = area of suction pipe.

$$\mathrm{A}_2 = \frac{25 \times 144}{8} = 450 \text{ square inches.}$$

This corresponds to a diameter of 24 in. nearly. The discharge pipe should increase until the velocity is considerably reduced. Suppose D = 4, and its area is A_3—

$$A_3 = 900 \text{ square inches,}$$

corresponding to a diameter of $33\frac{7}{8}$ in.

Besides friction, the only loss in this pump is—

$$L_2 + L_3 = \frac{u_1{}^2 + D^2}{2g}$$

$$= \frac{25 + 16}{64 \cdot 4} = \cdot 636 \text{ ft.}$$

The hydraulic efficiency $= 75$

$$= \frac{17}{22 \cdot 666}.$$

Therefore this calculation allows for a loss of 5·03 ft. of head by hydraulic friction.

CHAPTER XXXI.

The Fan.

Fans are used for exhausting air from passages such as those of a mine, the air being discharged into the atmosphere, or they force the air drawn from the atmosphere through the passages. In either case the volume of the air is only very slightly altered, and may be treated as constant and equal to the mean between that at suction and discharge. If Q is the number of cubic feet of air discharged per minute, and P the difference of pressure in pounds *per square foot* for suction and discharge, then the work done per minute is P Q foot-pounds, and consequently the useful horse power of the fan is

$$\text{H.P.} = \frac{P\,Q}{33000}.$$

The pressure P is obtained by a manometer in inches of water, and the volume Q by an anemometer. Some point in the passage through which the air passes is selected and divided by a grating into a number of rectangular areas; the quantity of air passing through each of these is measured during a given time, and the total for all the areas gives the discharge Q; to be more exact, we should say that the

anemometer is supposed to give the discharge, but in general largely exaggerates it. This has been proved by experiment by the Prussian Mining Commission in 1884.

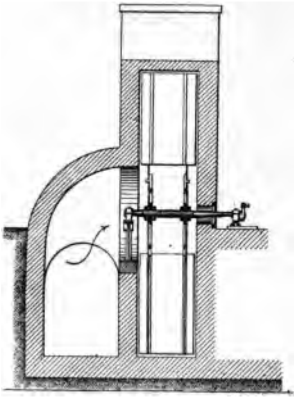


FIG. 142.

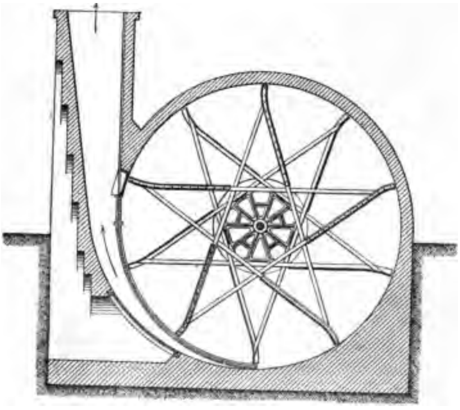


FIG. 143.

Before dealing with the subject of the fan itself, it will be as well to give an outline of these experiments. A spare gas holder was used for measuring the volume of the air; it

contained 70,634 cubic feet of air, and the discharge of air was measured by anemometers and Pitot tubes, and through circular and square orifices. The Pitot tube is merely a manometer with one end at right angles to and the other facing the flow of air, the depression being proportional to the square of the flow of air. The practical questions the

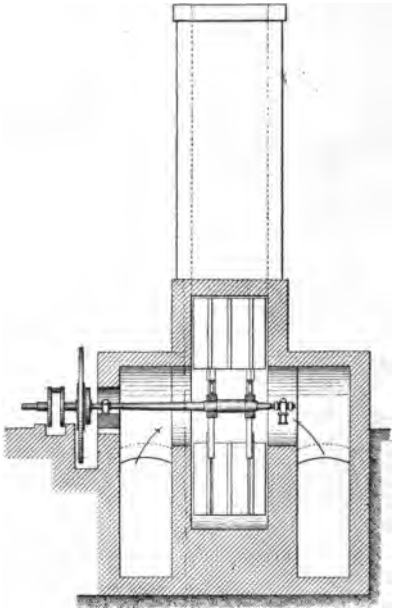


FIG. 144.

commission endeavoured to solve, by using this holder and causing the air to pass through a pipe, were the following :—

1. Do the formulæ generally used for standardising anemometers in a circular path in still air give correct results or not?

2. Can the Pitot tube be applied practically for measuring the speeds of air, and, if so, what formula should be used for calculating the speed and quantity of air?

3. May the fall in pressure between one side and the other of a thin orifice interposed in a pipe be used for calculating

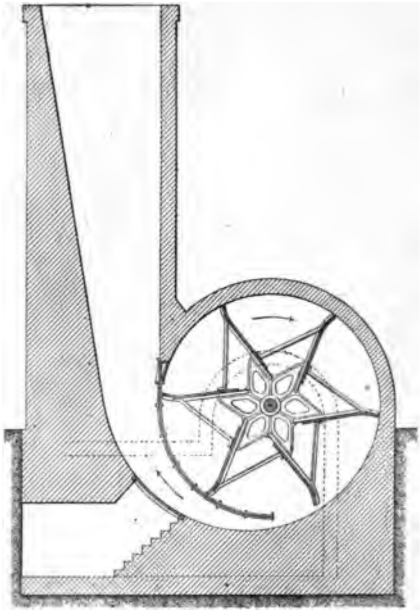


FIG. 145.

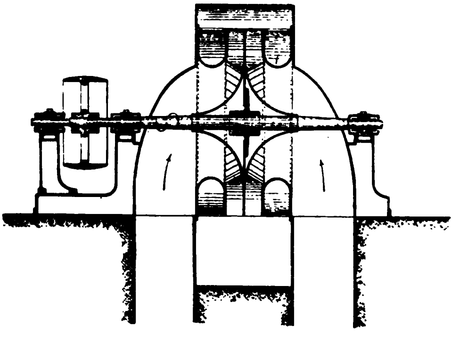


FIG. 146.

the quantity of air, and, if so, what formula should be applied?

4. What is the loss of head due to friction in regard to the length and diameter of the pipe used?

About eighty careful experiments were made, the cast-iron pipe being 14·3 in. diameter and 33 ft. long. For stopping and starting the anemometers quickly and accurately, an electrical arrangement was adopted, and the vertical fall was electrically determined. Experiments were made with water pressures of $2\frac{7}{8}$ in. and $4\frac{1}{2}$ in. of water. The density and temperature of the air were noted. A Pitot tube was used for measuring the dynamic pressures of air, not only

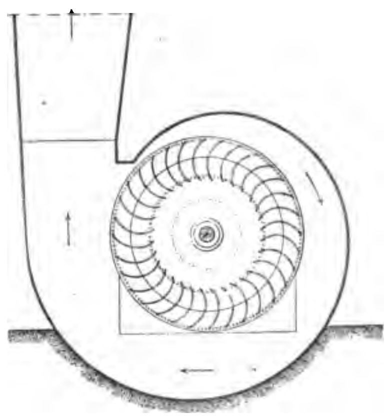


FIG. 147.

at the centre and at two-thirds of the radius distant from it, but also round the inner circumference of the pipe. The circular orifices used in these experiments measured 7·03 in. and 9·96 in. in diameter. The square orifice measured 6·26 in. along the side. The rectangular orifice was 9·17 in. by 4·45 in. The experimental coefficient determined for the circular orifices was ·64, and for the square orifices was ·61—that is to say, if Q is the number of cubic feet per second, A the area of the orifice in square feet, and H the head of air,

$$Q = C\,A\,\sqrt{2\,g\,H},$$

where C is the coefficient.

Four Casella anemometers were tested. The conclusions of the paper are the following: The Casella anemometers previously tested in the usual way at the end of a radius bar, and compared with direct measurement of air from the

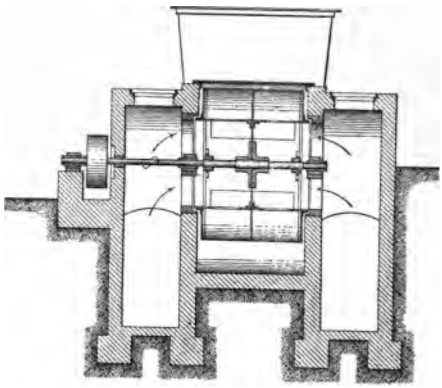


FIG. 148.

holder, showed errors, the excess ranging between 7 and 13 per cent. In the cast-iron 14·3 in. pipe the velocity increased from circumference to centre, and the mean speed was found

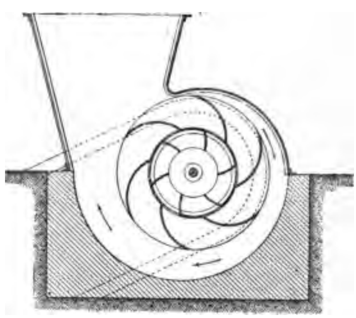


FIG. 149.

at two-thirds the radius from the centre. The resistance of cast-iron pipes was found to vary as the diameter of pipe raised to the power $1\frac{3}{8}$, as the square of the velocity, and as the density raised to the power $\frac{2}{3}$.

The formula for the Pitot tube is: Velocity of air in metres per second at zero Centigrade

$$= 4{\cdot}265 \sqrt{\frac{\text{pressure in mm. head of water}}{\text{density of air.}}}$$

This, reduced to the average temperature and density of the air, becomes $4 \sqrt{h}$, h being the head in mm. of water.

In the opinion of the author, the exaggeration is also largely increased in practice by the fact that the currents of air discharged from a fan have a variable velocity, and an anemometer gives a reading more nearly approaching to the square root of the sum of the squares of the varying

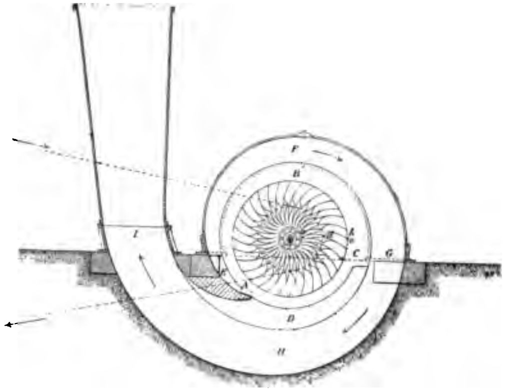


FIG. 150.

velocities than to their mean velocity. Thus, if during two successive seconds velocities existed proportional to 3 and 1, the mean velocity would be proportional to 2, but the reading of the anemometer would be nearer

$$\sqrt{\frac{3^2 + 1}{2}} = 2{\cdot}235,$$

the exaggeration being over 10 per cent; and as this exaggeration is in addition to that found by the Prussian Commission of 1884, due to uniform currents, a large exaggeration may be expected.

Before discussing the theory of the fan it will be as well to give a few examples of fans in use at the present day.

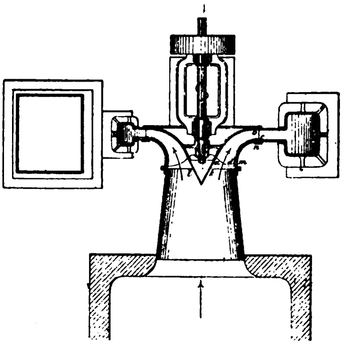


FIG. 151.

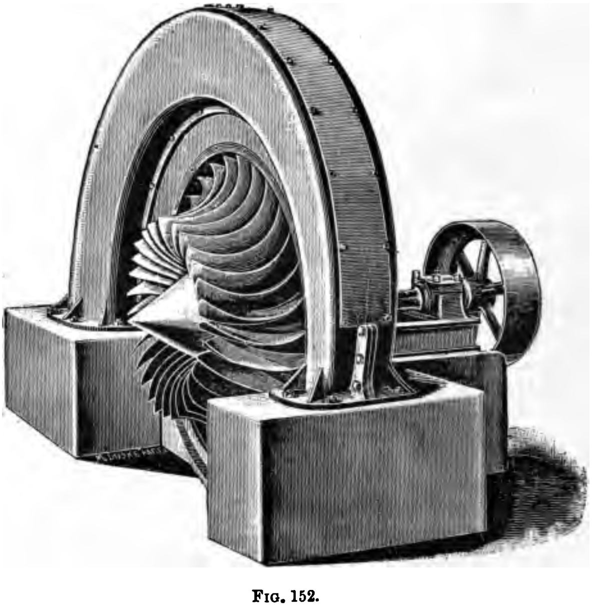


FIG. 152.

The earliest type of centrifugal fan was the Guibal, which, however, may be found ventilating many mines at the present day, both in this country and abroad. Its wheel is generally of considerable diameter, carrying a number of vanes, and enveloped over most of the circumference, and allowing the air to escape by a single opening, regulated by a shutter to suit the orifice of the mine. The air enters the eye, and by its centrifugal action it reaches the circumference and passes out at the chimney. The vanes of Guibals are sometimes plane and inclined in the opposite direction to that of rotation, but are usually curved near the outer extremity until they become radial. Their number is variable, and lies between six and ten for sizes varying between 19 ft. and 40 ft. The width of the vanes parallel to

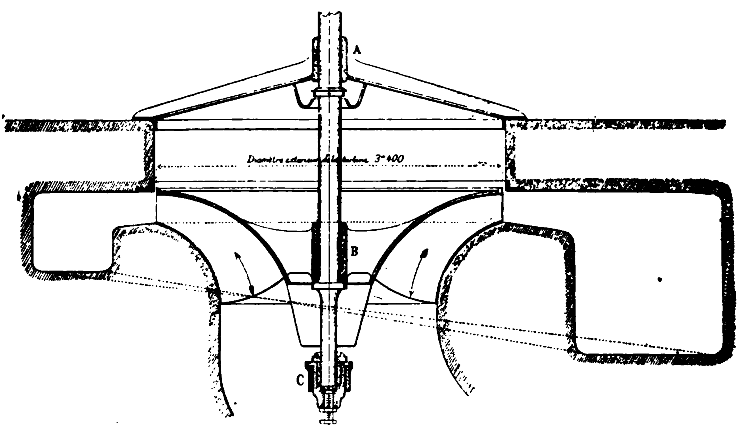


FIG. 153.

the axis lies between $4\frac{1}{2}$ ft. and 10 ft. for diameters of fan of 19 ft. and 40 ft. The object of the chimney is to reduce the velocity of the air, and consequently increase its pressure. Two examples of Guibals are shown in figs. 142, 143, 144, and 145, the former being 12 metres in diameter, or 39·3 ft., and the latter 5·8 metres, or 9 ft. diameter. In figs. 146 and 147 are shown two sections of the Ser ventilator, designed in 1878 by Prof. Ser, of the Ecole Centrale of Paris, the theory of which is published in the Mémoires de la Société des Ingenieurs

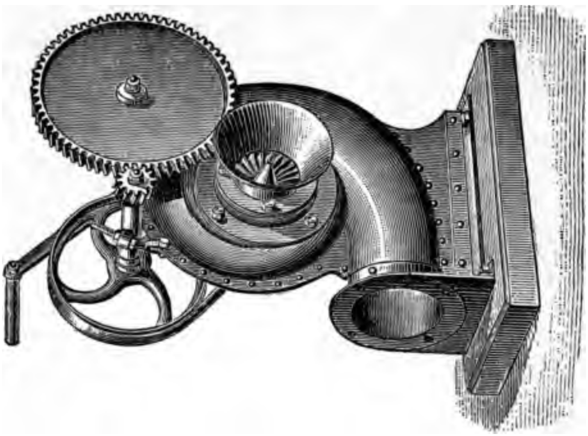


FIG. 155.

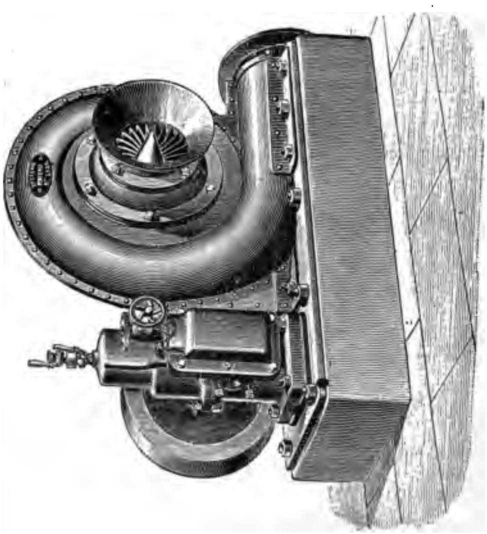


FIG. 154.

Civils for 1878. It consists of a circular plate fixed to the shaft and carrying on each side 32 curved vanes, each of which is a portion of a cylindrical surface whose generatrices are parallel to the shaft, and whose transverse section is circular. Their width is constant, and they are

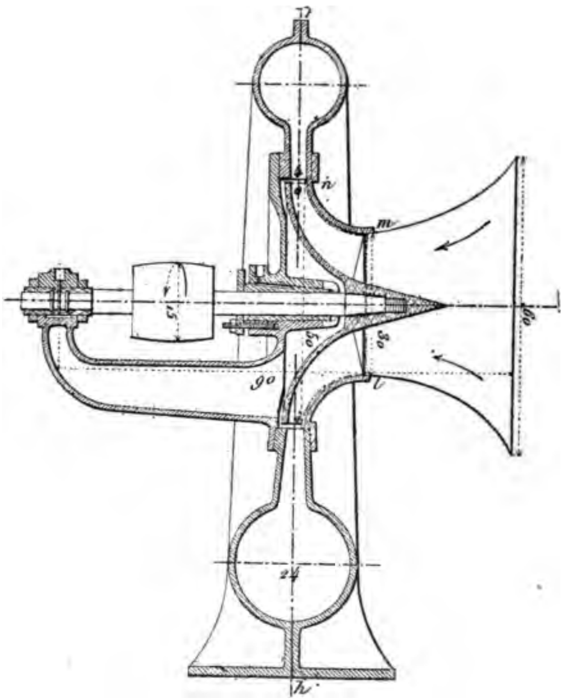


FIG. 156.

so arranged that inflow takes place without shock, and that the air is discharged from the fan so that the direction of its relative velocity makes an angle of 45 deg. with the tangent to the outer periphery. The air enters on both sides of the fan, and after passing through it enters a volute, which conducts it to an expanding chimney, from which it escapes into the atmosphere. The volute is so designed

that there is as little loss of energy as possible at entry from the circumference of the fan and while passing through it, and the sides of the chimney are inclined at not more than 1 in 8, to avoid the loss due to sudden enlargement of passage. Ser ventilators have a small diameter, from 1·4 metres, or 4·6 ft., to 2·5 metres, or 8·2 ft.

The Capell fan is shown in figs. 148 and 149. It is formed of two fans, one outside the other, having the same axis and revolving at the same angular velocity. The first consists of a drum of steel plate, closed if there is a single eye on one of

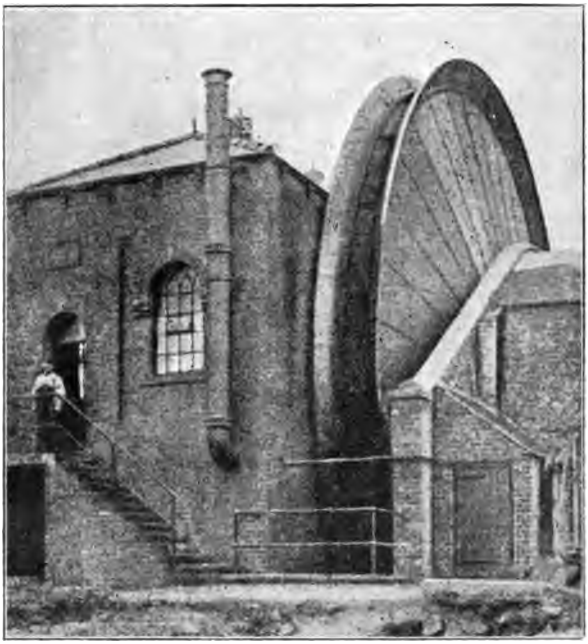


FIG. 157.

its side faces, and open on the other to receive the air at inflow; its diameter is equal to that of the eye. The cylindrical surface contains six openings, in general rectangular and spaced at equal intervals, whose area is less

than the cylindrical surface of the drum, but equal to that of the eye at least. Six vanes of steel plates, curved and cylindrical, directed in passing from the centre to the circumference in the direction opposite to that of rotation, end at one of the sides of the openings in the drum; these end at the interior of the drum. The second part of the wheel is larger than the first, and is completely closed at the sides by two annular discs of steel plates; the cylindrical surface is completely open, and through it the air leaves the

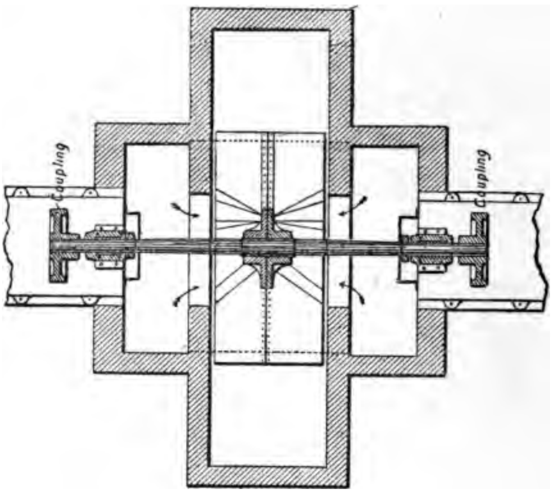


FIG. 158.

fan. Between the two discs are six vanes, curved in the opposite direction to that of rotation, in a manner very clearly shown in fig. 149. A spiral chamber and rapidly-expanding chimney form the casing. They are constructed up to 20 ft. for mine ventilation.

The Rateau ventilator is illustrated in figs. 150 and 151 in sectional elevation and plan. It is designed by Professor Rateau, of the Ecole St. Etienne, France. It is generally constructed with one eye, and the fan is carried at the end of the shaft, which is supported on two bearings fixed to a stone foundation. Although the fan is at the end of the shaft, its vanes are curved so that its centre of gravity comes

over the outer end of the bearing, so that the overhang of the total weight is small. The wheel consists of a casting formed by the revolution of a circular arc ab about the axis of rotation, to which are fixed by means of angle irons thirty steel vanes of special form, fashioned in the hydraulic press. At the centre of the eye there is a cone, and at its outer circumference a frustrum of a cone, by means of which the air is guided to the eye, and the flow is made practically uniform over its whole surface. The curve ad is a circle,

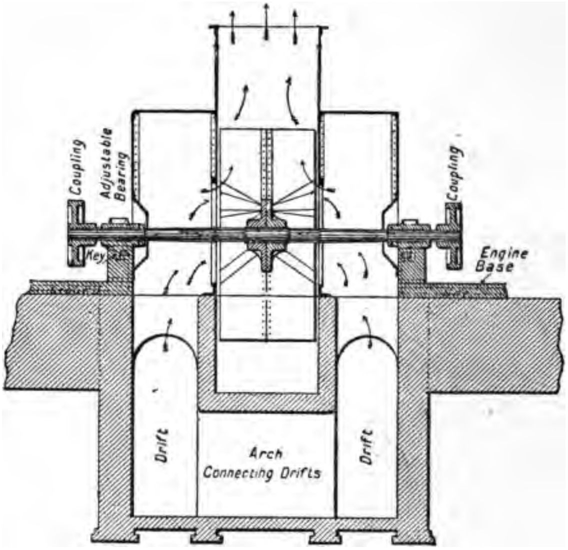


FIG. 159.

and dc a quarter of an ellipse. The vanes are very rigid on account of their curvature; the air is received without shock into the fan, and discharged with a high tangential velocity into the diffuser, the sides of which are plane and inclined to one another at about 7 deg. The diffuser is not cylindrical, but its outer circumference is spiral in form, the increase of the radius being proportional to the angle θ from the beak A; from the diffuser the air flows into a volute whose section S is calculated from a formula,

$$S = a\theta + b\theta^2,$$

and not $S = a\theta$, as is usual with diffusers having a cylindrical circumference.

Finally, the air enters the chimney, whose sides are inclined at about 7 deg. The volute is constructed partly in iron and partly in brickwork. The chimney is of steel or masonry. Fig. 152 gives a perspective view of one of these ventilators with part of the casing removed. Although generally constructed with a horizontal axis, there are certain particular cases where it is advantageous to use a vertical axis, fig. 153. It is then possible to make the diffuser and volute and one side of the wheel case in masonry. The shaft is carried by a footstep bearing, containing means of adjusting its height, and it can be driven by bevel wheels, ropes, or direct by a steam engine having a vertical axis.

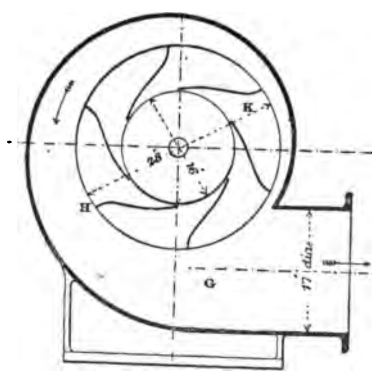


FIG. 160.

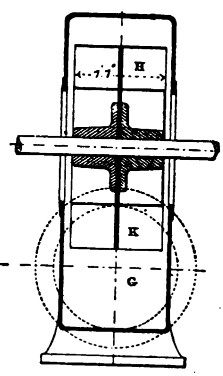


FIG. 161.

These fans are made with diameters of from 1 to 4 metres—that is, from 3·28 ft. to 13·1.

Small ventilators, driven by engine or by hand, are shown in figs. 154 and 155, and a sectional elevation of one is shown in fig. 156.

The Waddle open-running fan, fig. 157, is also largely used in this country. Its vanes curve backwards at the outer periphery in the opposite direction to that of rotation, and the curved rim is intended to reduce the velocity of the air at discharge, and so convert the kinetic energy of discharge from the fan into pressure energy, although the curvature

seems rather too rapid to effect this properly, considering the high velocity with which the air is discharged from the fan.

The Bumstead and Chandler fan, figs. 158 and 159, runs in a casing of spiral form. The blades are stated* to be of

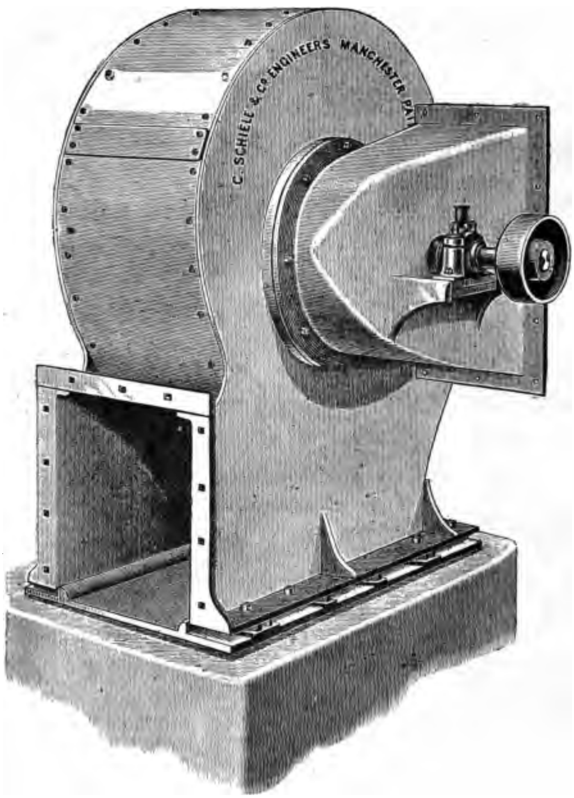


FIG. 160.

modified S form, curved forwards so as to cut into the air at entry, the velocity of the air being gradually increased until

* The *Engineer*, January 20th, 1893.

outflow from the fan. An évasé chimney is added to reduce the velocity of the air to as low a value as possible.

The Heenan and Gilbert fan, figs. 160 and 161, has vanes which meet the air edgeways at inflow and curve backwards

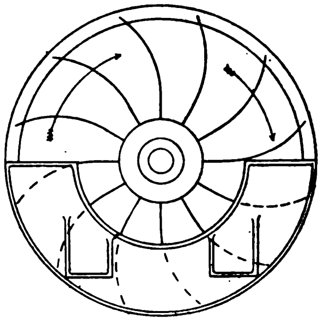


FIG. 163.

at first, but become radial at the outer periphery, their form being somewhat similar to that suggested by Prof. Rankine.

The majority of fans, however, especially those used in this country, have vanes curving in the direction opposite

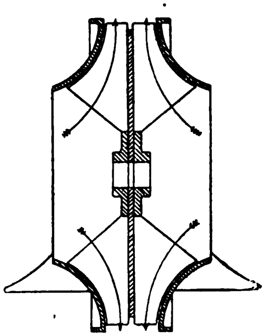


FIG. 164.

to that of rotation. The Schiele fan, which is largely used in England, is an example of this. An outside view is shown in fig. 162, and a side elevation and sectional elevation in figs. 163 and 164.

FIG. 165.

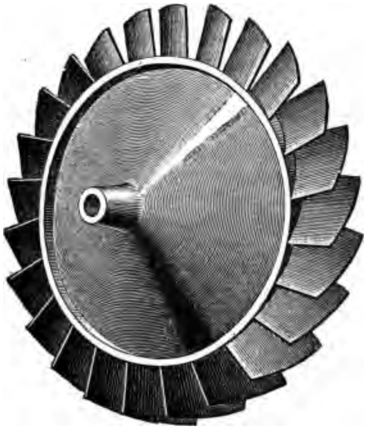
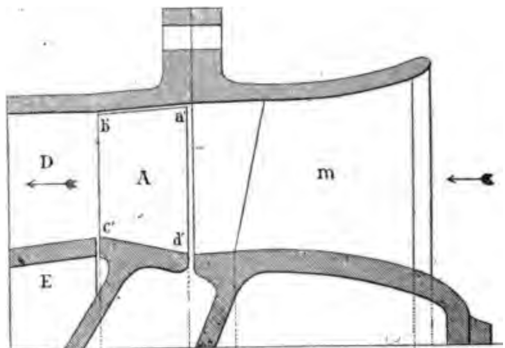


FIG. 166.

In the Rateau screw fan the general direction of flow is axial. The first type is shown in figs. 165, 166 in perspective, and 167, 168 in section. It will be seen that there are guide vanes *m*, *m*, fig. 168, which give the air a tangential motion before reaching the fan, but in an opposite direction to the direction

FIG. 167.



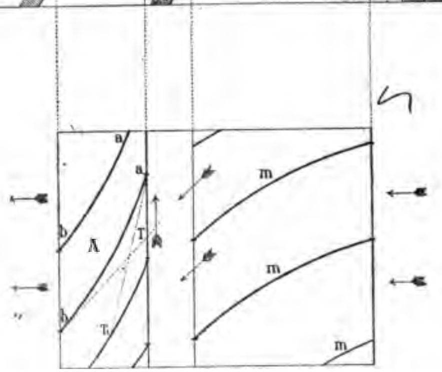
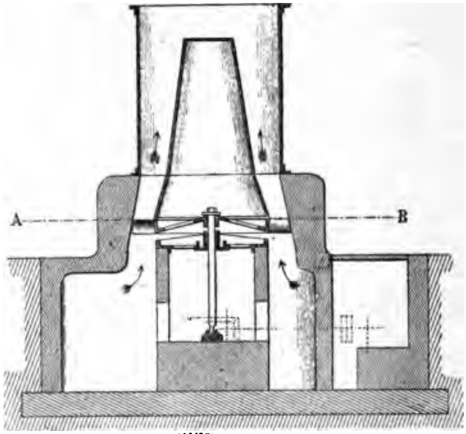


FIG. 168.

of rotation. The object of the revolving wheel is to reduce this tangential velocity to zero and discharge the air axially, so that if w_2 is the tangential velocity of the air, and c is the mean velocity of the wheel, the work done per pound is $\frac{c\,w_2}{g}$.

In fig. 167 the vane is a^1, b^1, c^1, d^1, and a^1, b^1 has a very slight clearance from the casing, which is made of masonry for large sizes and cast iron for small. The inflow is at a^1, d^1, and the discharge at b^1, c^1. The vanes are helical surfaces of variable pitch, whose generating line is perpendicular to and passes through the axis; $a\,b$, fig. 168, is a circular arc.

FIG. 169.



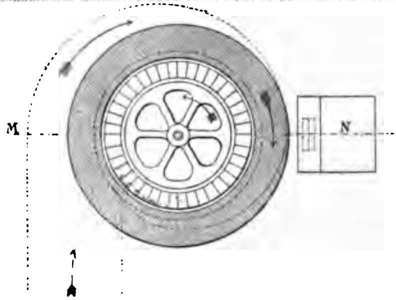


FIG. 170[1]

Theory demands that the air should be given a slightly centripetal motion, and the casing is therefore slightly coned at a^1, b^1.

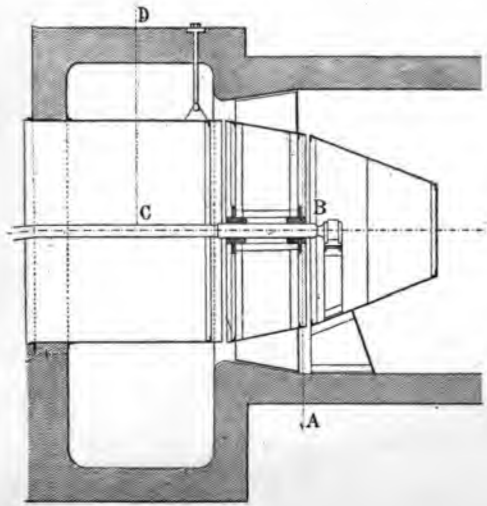


FIG. 171.

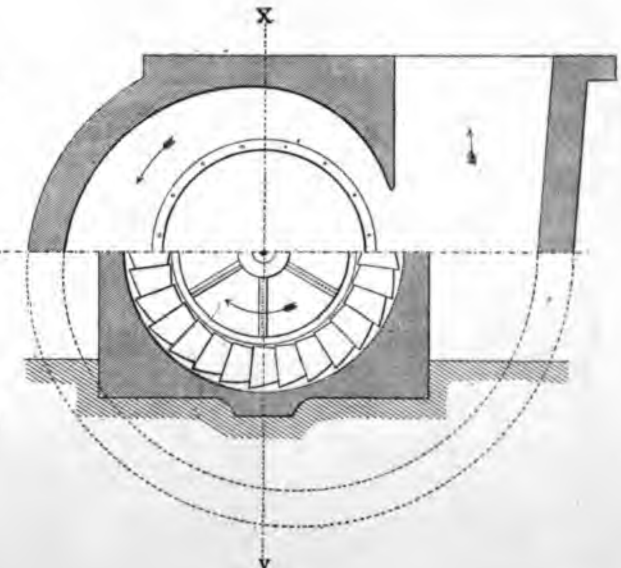


FIG. 172.

The number of vanes is from twenty-four to thirty-six; they are made of steel, and fastened to the rim by angle irons, or in small sizes are cast in.

A second form of fan is shown in sectional elevation and plan in figs. 169 and 170. The inflow passage is a volute, which gradually decreases in section, and gives the air a tangential velocity, which is reduced to zero in the fan. The discharge pipe is given an increasing section by an internal frustrum

FIG. 173.

of a cone, fig. 169, so that the kinetic energy of the air is partially converted into pressure. In figs. 171 and 172 this arrangement is reversed, the air entering by the conical passage, and being discharged through the fan into a spiral passage which enlarges in section, and so reduces the velocity of the air. Fig. 173 shows the manner in which one of these fans may be driven by an electro-motor.

Such fans as these are very well suited to confined situations, unsuitable for the ordinary centrifugal type of fan. Their mechanical efficiency, as given by the anemo-

meter, is about 60 per cent, and their manometric power may be varied, to suit the circumstances, from 15 to 75 per cent. These fans are made by Messrs. Biétrix and Co., St. Etienne, and this description is taken from their illustrated catalogue.

CHAPTER XXXII.

The Theory of the Fan.

The theory of the fan is the same as that of the centrifugal pump, although the latter discharges an incompressible substance; for the greatest water gauge against which a fan works is about 6 in., and as the water barometer is about 34 ft., this corresponds to a compression of $\frac{1}{68}$ of its original volume. If, therefore, we take the mean volume of air passing through the fan, the variation of volume on either side is not more than $\frac{1}{136}$ of the mean. In exact experiments on fans, the density of the air must be calculated by means of the barometer and hygrometer; but if h be the water gauge in inches, and H the equivalent head of air in feet, then

$$H = \frac{10000}{144} h,$$

very nearly, at moderate heights above the sea level, and may be used for experiments where great accuracy is not required.

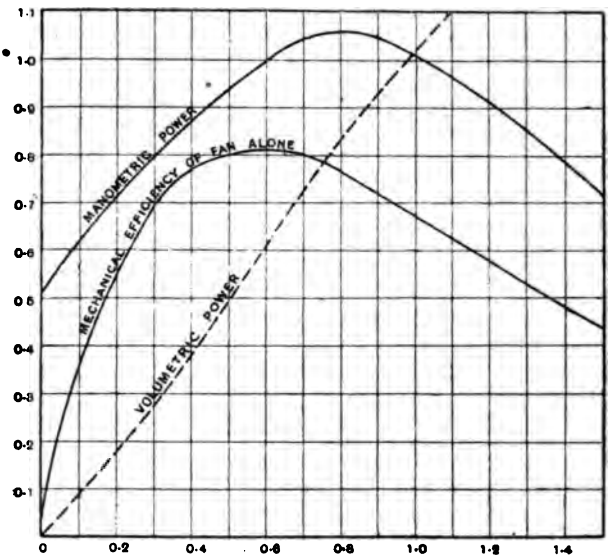
The work done by the fan per pound of air is $\frac{c_1 w_1}{g}$, where c_1 is the circumferential velocity of the fan, and w_1 the tangential velocity of the air at discharge from the fan, and the volute and diffuser may be proportioned by exactly the same rules given for the centrifugal pump. Where, however, space is of no consequence, as at a mine, and a large diffuser can be used, the vanes of the fan should be curved forward, so as to make w_1 greater than c_1, as in the Rateau ventilator, in which the angle of discharge is 45 deg. from the radius, and

$$\therefore w_1 = c_1 + u_1,$$

where u_1 is the radial component of the velocity of discharge; but where, however, space prevents a large diameter, and a diffuser cannot be used, the vanes must curve backwards, otherwise the efficiency will be low.

The apparent experimental efficiencies of various fans are very misleading, as they are mostly obtained with the anemometer, which we have already stated largely exaggerates

CHARACTERISTIC CURVES OF RATEAU FANS.



REDUCED ORIFICES.

FIG. 174.

the discharge, owing mainly to the inertia of the anemometer and the irregular motion of the air. We cannot therefore use the equation

$$\frac{c_1 w_1}{g} = \frac{H}{\eta},$$

which we have already obtained for the centrifugal pump, as the values of η, the efficiencies obtained by experiment, are much too high. The Pitot tube is far preferable to the anemometer, and, although it exaggerates the discharge, its errors are not so great, and by a proper arrangement of

apparatus they may be very much reduced. Experiments on a number of fans with the Pitot tube were made by Mr. Bryan Donkin, and are given in the Minutes of the Proceedings of the Institution of Civil Engineers, vol. cxii., and the lowness of the efficiencies goes a long way to prove their accuracy.

Although it might be used with advantage in the case of a centrifugal pump, the terms orifice and equivalent orifice are only used in the case of the centrifugal fan. The latter is due to M. Murgue, and is $\frac{Q}{\cdot 65 \sqrt{2gH}}$; while the former is $\frac{Q}{\sqrt{gH}}$, and is used by Professor Rateau. Q is here the number of cubic feet or metres of air discharged per second, and H is the head of air in feet or metres. The latter is more scientific than the former, because this assumes an arbitrary coefficient of contraction. Experiment and theory both show that if a fan is forcing air through passages, that $Q^2 \propto H \propto c_1^2$, and hence it is convenient in diagrams of a fan's working to take orifices, or equivalent orifices, as abscissæ. Two diagrams may be drawn, one showing the variation of mechanical efficiency, and the other of the manometric power or efficiency (the former being the better, the latter the more usual term), with the orifice.

Manometric power is $\frac{gH}{c_1^2}$, and measures the rapidity of rotation required to produce a given draught in the mine. The diagrams of a Rateau ventilator are given in fig. 1, the abscissæ being reduced orifices, so that experiments for a number of similar fans may all be combined in one. The term "reduced orifice" is also due to Professor Rateau, and is $\frac{Q}{R^2 \sqrt{gH}}$, where R is the external radius of the fan centre.

The general equation of a fan is

$$c_1^2 + p c_1 Q - q Q^2 - r H = o;$$

and if μ is the manometric power, this may be thrown in the form

$$\frac{1}{\mu} + \frac{s \cdot O}{\sqrt{\mu}} + t \cdot O^2 + u = o;$$

where p, q, r, s, t, u are constants, and O is the orifice or equivalent orifice. As a proof of this, we add a comparison

of experiments taken from Mr. Bryan Donkin's paper, above mentioned, and calculations from equations similar to the second of the above.

TABLE I.—FAN NO. VIII. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice. Square feet.		Actual Manometric power. Per cent.		Calculated Manometric power. Per cent.
0	......	59·0	......	59·0
0·1	......	52·5	......	54·5
0·2	......	43·0	......	43·5
0·3	......	32·0	......	33·1
0·4	......	24·0	......	24·7
0·5	......	19·0	......	18·6
0·8	......	9·5	......	9·0
1·0	......	6·6	......	6·1
1·5	......	3·0	......	2·87

The equation to the above fan is

$$\frac{1}{\mu} - \cdot 136 \frac{O}{\sqrt{\mu}} - 14{\cdot}18\, O^2 - 1{\cdot}69 = 0.$$

TABLE II.—FAN NO. VI. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice in square feet.		Calculated manometric power. Per cent.		Actual manometric power. Per cent.
0	......	60·0	......	60·0
0·1	......	57·5	......	58·0
0·2	......	54·1	......	54·0
0·3	......	50·0	......	50·0
0·4	......	40·4	......	43·0
0·5	......	36·25	......	36·25
0·8	......	20·7	......	22·0
1·0	......	14·25	......	15·0

The equation to the above fan is

$$\frac{1}{\mu} + \cdot 192 \frac{O}{\sqrt{\mu}} - 5{\cdot}85\, O^2 - 1{\cdot}666 = 0.$$

TABLE III.—FAN No. X. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice in square feet.	Calculated manometric power. Per cent.	Actual manometric power. Per cent.
0	57·0	57·0
0·1	63·0	60 0
0·2	59·8	59·8
0·3	51·0	52·0
0·4	41·6	41·6
0·5	33·0	33·0
0·8	17·7	17·85
1 0	12·1	16·0
1·5	6·34	7·5

The equation to the above fan is

$$\frac{1}{\mu} + 2{\cdot}43\frac{O}{\sqrt{\mu}} - 13{\cdot}55\,O^2 - 1{\cdot}755 = 0.$$

TABLE IV.—FAN No. XI. MR. BRYAN DONKIN'S EXPERIMENTS.

Equivalent orifice in square feet.	Calculated manometric power. Per cent.	Actual manometric power. Per cent.
0	28·5	28·5
0·1*	32·8	26·0
0·24	19 0	19·0
0·3	14·5	15·5
0·4	9·47	10·5
0·5	6·5	6·5
0·8	2·78	2·8
1·0	1·8	2·0
1·5	·08	1·0

The equation to the above fan is

$$\frac{1}{\mu} + 10{\cdot}05\frac{O}{\sqrt{\mu}} - 126{\cdot}4\,O^2 - 3{\cdot}51 = 0.$$

If we can, for a given type of fan, find the real value of η, we can then proceed to design the fan in the same way as a

* The smallest equivalent orifice at which a test was made, except zero orifice, was ·24 square feet.

centrifugal pump. In our opinion an efficiency of 60 to 66 per cent may be assumed for a well-designed fan, in the equation

$$c_1 w_1 = \frac{g\,H}{\eta}.$$

In the Rateau ventilator the values of u_1, the radial component of overflow from the fan, and u_2, the axial velocity of inflow, are, when maximum efficiency is obtained, about $\cdot 58 \sqrt{g\,H}$, and with an efficiency of $\frac{2}{3}$ this gives us—

$$\frac{g\,H}{c_1(c_1 + u_1)} = \frac{2}{3}.$$

$$\frac{g\,H}{c_1^2 + \cdot 58\, c_1 \sqrt{g\,H}} = \frac{2}{3}.$$

$$c_1^2 + \cdot 58\, c_1 \sqrt{g\,H} - \tfrac{3}{2} g\,H = o.$$

$$c_1 = \cdot 97 \sqrt{g\,H},$$

and the manometric power is

$$\frac{g\,H}{c_1^2} = 1\cdot 06.$$

CHAPTER XXXIII.

The Hydraulic Works at Niagara.

Since concluding the subject of the turbine, we have obtained descriptions of the above works. As they contain the most powerful reaction turbines in the world, we should consider these articles incomplete if we omitted an account of them.

Sir William Siemens is reported to have said that if all the daily output of coal in the world could be used in making steam to drive pumps, it would barely suffice to pump back the water flowing down Niagara River. Many schemes have therefore been brought forward, discussed, and discarded for various reasons, one of which was that the picturesque aspect of the place would have been interfered with by them; and this was right, for there are few things on this earth more valuable to mankind than the beauties of nature. Some forty years ago the so-called hydraulic canal, operating factories about one-quarter of a mile below

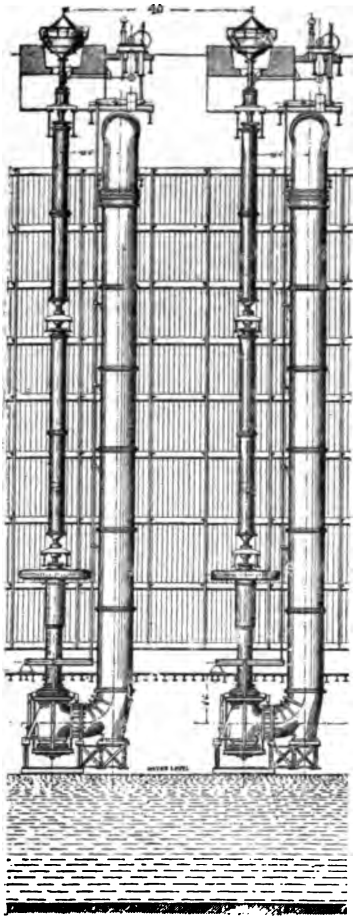


FIG. 175.

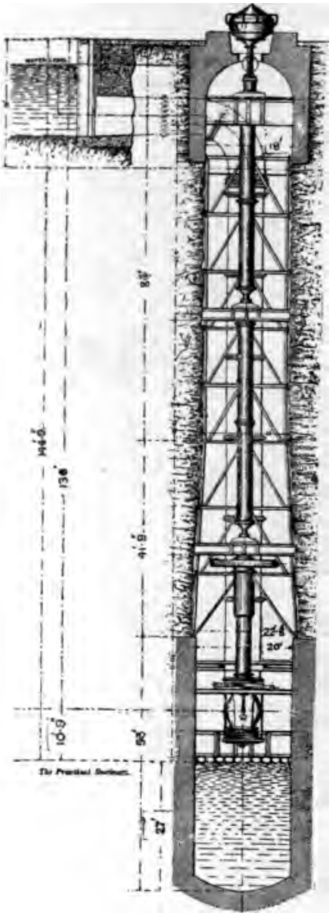


FIG. 176.

the Falls, was built. From this canal not more than 6,000 horse power is obtained. The present Niagara Falls Power Company have secured a strip of land along Upper Niagara River, and extending some distance back along the shore. This will furnish sites for factories, to which power will be supplied. The utilisation of the power will be left to the owners of the factories, the company generating and transmitting it to any desired points by means of electricity. The preliminary work was the examination by the President, Mr. E. D. Adams, and Mr. Coleman Sellers of the principal

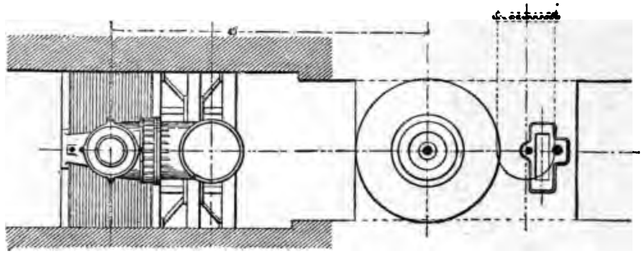


Fig. 177.

methods of utilising a fall of water. This necessitated an examination of plans in operation in Europe, especially in Switzerland. The vast volume of water at the company's disposal makes the case very different, and the study of plans for the development of 1,000 horse power is very little guide where it is necessary to develop over 100,000 horse power. The quantity of water is also practically constant, whereas in Europe, where falls are high, the flow is generally variable. In addition, a vitally essential part of the question was the transmission of this power after its development. The Niagara Commission was therefore formed, composed of men of eminent talent, who were to invite plans from the most prominent and competent engineers of the world for the solution of the above problems. The method most favourably considered for the generation of the power was by turbines. There were fourteen different methods, one of which was the compression of air to a pressure of 150 lb.; and the remainder included reaction and impulse turbines, with radial and axial flow, various devices being suggested to neutralise the pressure of the tremendous head, and to overcome the friction of the shafting caused by its weight and that of the wheel; several plans proposed forcing fluid

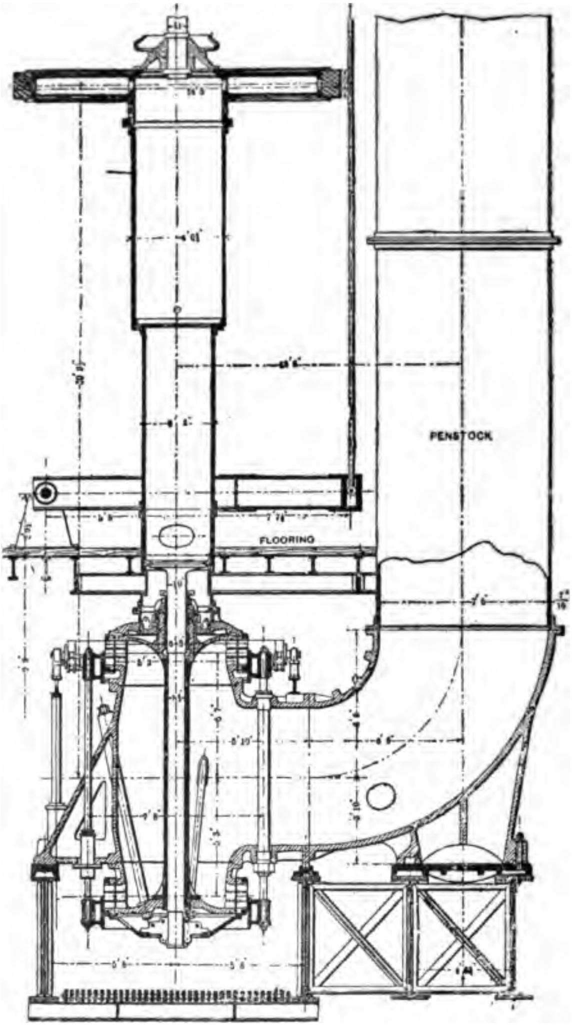


FIG. 178.

between the bearing surfaces, and converting the bearings into fluid bearings. Others proposed to construct the wheels so that the water not only turned them, but also served to lift the shaft, and thereby relieve the bearings of undue friction.

The turbines accepted were those designed by Messrs. Faesch and Piccard, of Geneva, and the contract was let to J. P. Morris and Co., Philadelphia. The general arrangement of these wheels in relation to the shaft in which they are placed will be seen from figs. 175, 176, and 177. Each

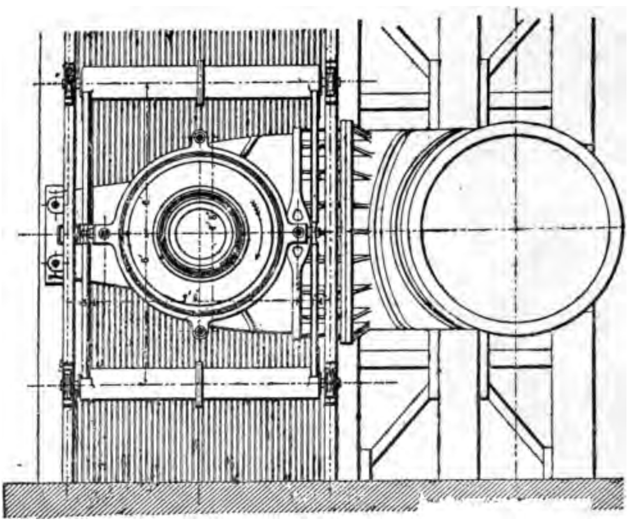


FIG. 179.

turbine operates under an average head of 136 ft., and consists of two wheels to each shaft. Each turbine develops 5,000 horse power at 250 revolutions per minute, with a volume of water equal to 430 cubic feet per second. The shaft is vertical, and drives a dynamo direct at the top of the shaft. The wheels are of bronze, and are shown in figs. 178 and 179, while the lower wheel is shown to a larger scale in figs. 180 and 181. They are reaction wheels, although the fact that they discharge into the atmosphere might lead one at first to suppose that they were impulse turbines. This will be

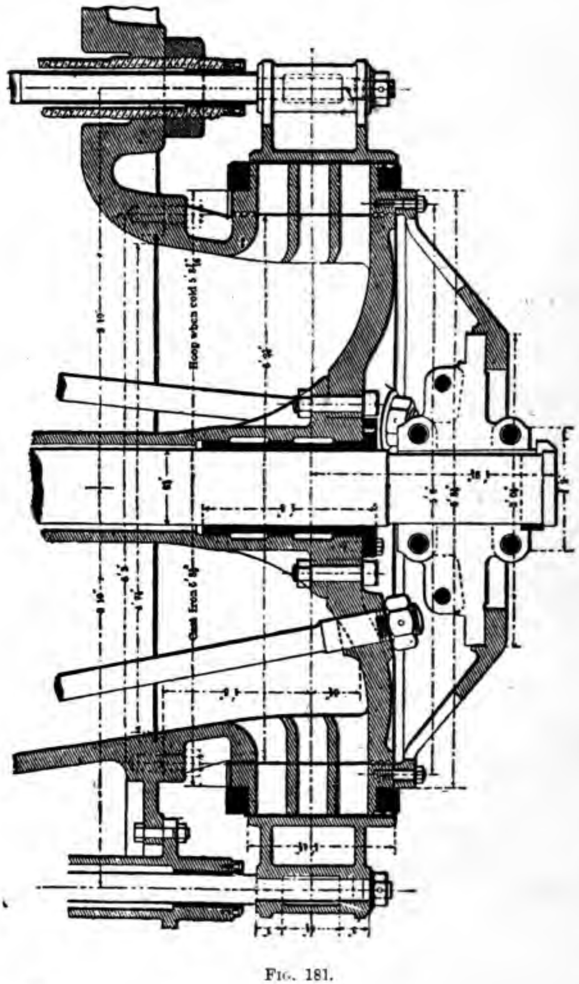


Fig. 181.

evident to anyone who has read the previous articles, for the velocity of whirl w_1 that would be obtained if the

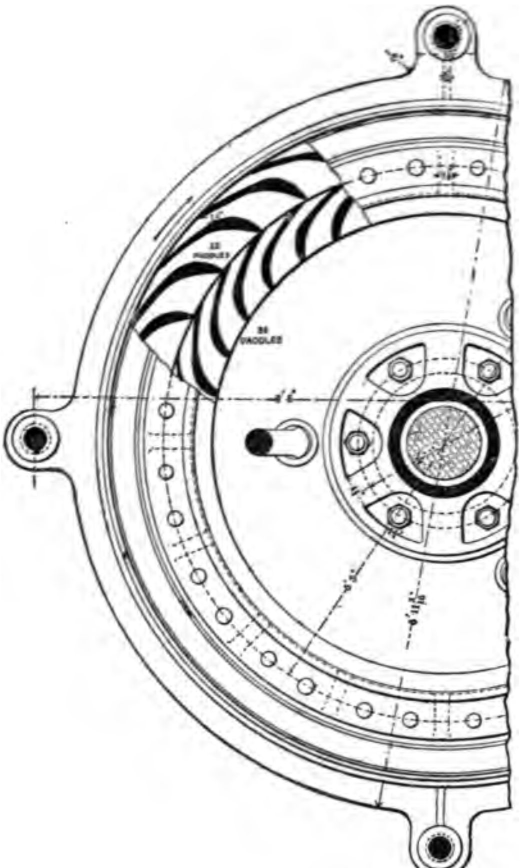


FIG. 180.

discharge took place from the guide passages against the pressure of the atmosphere could not exist with the high

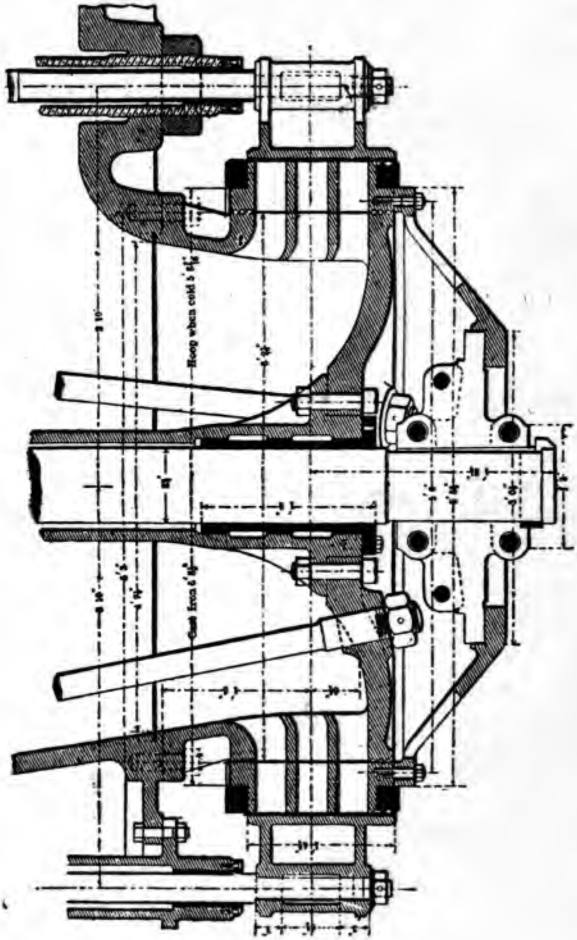


FIG. 181.

value c_1, which is more than 68 ft. per second. There are 36 guide and 32 wheel vanes. A steel pipe, 90 in. diameter, conducts the water from the canal to a chamber between the two guide wheels. The main shaft is made of steel tubes, 38 in. diameter, and 11 in. solid shafts are interposed at the points where the guide bearings are placed. This lightens and increases the rigidity of the shaft, and also reduces the necessary number of guides. A heavy flywheel

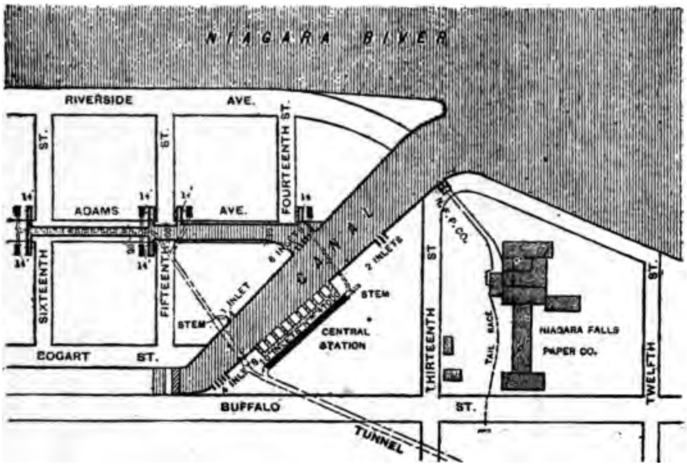


FIG. 182.

of forged iron, weighing 10 tons, with a diameter of $14\frac{1}{2}$ ft., is attached to the shaft, as shown in fig. 178; its peripheral speed is 11,000 ft. per minute. The upper bearing of the shaft is completely relieved of its load when the turbine is in motion, because the pressure of the water acts upon the lower surface of the upper wheel. This method of balancing Fourneyron turbines is not new, however.

The speed is regulated by ring sluices, not between the wheel and guides, but outside the wheel. The division of the wheel into three parts makes this an economical method. This sluice is raised or lowered by means of a system of rods and levers, clearly shown in the figures, which are controlled by a very sensitive governor of the indirect-acting class. The speed varies between a maximum of $\frac{1}{2}$ per

cent at the normal rate of working, and about 3 per cent if the capacity of the turbine is increased or diminished suddenly by one quarter. This sensitiveness is necessary for an electric installation.

A tunnel, commenced October 4, 1890, has been made to carry off the water from the wheels. It is 6,700 ft. long, with a section of 490 square feet, and a hydraulic gradient of about 7 ft. in 1,000, and discharges its water into the river again below the Falls, and near the Suspension Bridge. The velocity of flow is given by Professor Forbes as 25 ft. per second, which we believe to be unprecedented. Ordinarily, about 8 ft. per second is considered as the extreme limit in brick-lined sewers; indeed, a leading waterworks engineer allows only about 3 ft. as a maximum in lined channels. The lining here is made of 16 in. of hard brick, this having been found necessary because the rock passed through disintegrated or crumbled quickly when exposed to the air. The brick and hydraulic cement used had united to form a mass as solid as rock in some specimens taken from the tunnel.

Fig. 182 is a sketch of the general plan of the works. The Niagara Falls Paper Co. is already in operation.

CHAPTER XXXIV.

Hydraulic Buffers.

The fact that work must be done to cause water to flow with a given velocity may be utilised to arrest the motion of a rapidly-moving mass, such as a gun, or a train at a terminal station when the ordinary brakes have failed to act, or the driver has miscalculated their power.

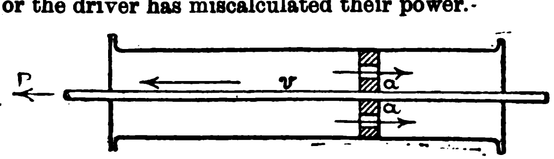


Fig. 183.

Fig. 183 shows a cylinder in which is a piston with passages *a a*, by means of which the water on the left-hand

side can be allowed to flow to the right when the piston is pulled to the left by means of a force P.

Let v be the velocity of the piston at any instant, and m the ratio of the area A of the piston to that of the orifices, allowing for contraction of discharge through the latter. Then the absolute velocity of flow is $(m - 1)\,v$, and the number of pounds of liquid passing the piston during the indefinitely small time t is $\mathrm{D\,A}\,v\,t$ where D is the weight of a cubic foot of the liquid, which may be water or oil. During this time the piston has moved a distance $v\,t$. Let P be the whole pressure thereon ; then the work done

$$= \mathrm{P}\,v\,t.$$

This equals the kinetic energy produced, and therefore—

$$\mathrm{P}\,v\,t = \mathrm{D\,A}\,v\,t\,.(m - 1)^2\,\frac{v^2}{2g},$$

and

$$\mathrm{P} = \mathrm{D\,A}(m - 1)^2\,\frac{v^2}{2g}.$$

Let F be the constant force of friction, V the initial velocity, and E the initial energy of the body to be arrested, which is all absorbed by the buffer, and finally converted into heat. Also let P_1 be the maximum value of P, supposing m constant. Then—

$$\mathrm{P}_1 = \mathrm{D\,A}(m - 1)^2\,\frac{\mathrm{V}^2}{2g}$$

and $\mathrm{P}_1 + \mathrm{F}$ is the maximum retarding force.

It is clear that the pressure tending to burst the cylinder will be least when m is made to vary so that P is constant ; and there are several devices which reduce the area of the orifices as the recoil proceeds. Theoretically, then, $(m - 1)\,v$ should be constant, and if l is the length of stroke, then—

$$\frac{\mathrm{W\,V}^2}{2g} = (\mathrm{P} + \mathrm{F})\,l$$

$$= \left\{\frac{\mathrm{D\,A}\,(m_1 - 1)^2\,\mathrm{V}^2}{2g} + \mathrm{F}\right\} l,$$

so that if F, W, and V are known, and any two of the three quantities A, m_1, l assumed, the third may be calculated. In this last equation, m_1 represents the ratio of A to the effective area of the orifices at the commencement of the retardation. If s be the distance from the end of the stroke, then—

$$v^2 = 2g\,\frac{\mathrm{F} + \mathrm{P}}{\mathrm{W}}\,s\,;$$

and since $$(m-1)\,v = \text{a constant},$$

$$\therefore (m-1) \text{ is inversely proportional to } \sqrt{s}.$$

The simplest method of reducing the orifices is to have two keys parallel to the axis of the cylinder, which increase in height from beginning to end of stroke. Two corresponding slots in the piston form the orifices, and plates are held upon the piston, so that the sizes of the slots may be adjusted by experiment until the desired length of stroke is obtained. In the Minutes of Proceedings of the Institution of Mechanical Engineers, for 1886, there is a description by Mr. Alfred Langley of a hydraulic buffer stop for terminal stations. The above arrangement of keys is adopted in this. The length of stroke allowed is 4 ft.; the diameter of cylinder is 12 in., with a piston rod of steel $3\frac{3}{4}$ in. diameter. The piston is turned an easy fit, and the clearance space between its circumference and that of the cylinder gives an area of ·38 square inch.

When the buffers are fully out, the combined area of slots and clearance are 5·43 square inches, decreasing to 1·4 square inch when the piston has made 34 in. of stroke. We cannot say what the coefficient of contraction is, although this could readily be found by experiment, The keys are 3 in. wide, and project $\frac{9}{16}$ in. into the cylinder at the beginning of the stroke, tapering to $1\frac{11}{16}$ in. at the rear end. The slots are $1\frac{11}{16}$ in. deep.

A train going at eight miles an hour is brought to rest before the stroke of 4 ft. is completed, without any damage to train or buffer.

The buffers have the advantage of giving no recoil, and in this are superior to spring buffers. They are drawn forward by a weight, fastened by a chain to the tail end of the rod, which passes over a pulley, and in order to reduce the sudden stress on the chain when motion commences, a seating of indiarubber is placed between the bottom weight and the bolt that holds it. This has been found to be the best plan for bringing the piston forward.

The calculations required when the area of the orifices is constant require differential and integral calculus. Where this has been unavoidable in these articles, we have used it, but as there is no difficulty in making the orifices variable, and it is better to do so, there is no advantage in treating the case when they are not so.

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